SMAUG-T: the Key Exchange Algorithm based on Module-LWE and Module-LWR

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Abstract. This paper introduces SMAUG-T, a lattice-based post-quantum key exchange algorithm submitted to Round 2 of the Korean Post-Quantum Cryptography Competition (KpqC). SMAUG-T is designed by merging SMAUG and TiGER from the KpqC Round 1. The algorithm is based on the hardness of the MLWE and MLWR problems defined in the module lattice and using sparse secret chosen by SMAUG. Along with the original SMAUG parameter sets, we introduce a TiMER (Tiny SMAUG using Error Reconciliation) parameter set suitable for the IoT environment. In terms of size, SMAUG-T achieves ciphertext and public key that is up to 14% and 19% smaller than Kyber, respectively. From a performance perspective, encapsulation demonstrates high efficiency, achieving up to 60% faster than Kyber in the constant-time C implementation and up to 70% in the AVX2 implementation.

Keywords: Lattice-based Cryptography · Post-Quantum Cryptography · Key Encapsulation Mechanism · Module Learning With Errors · Module Learning With Roundings.

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Changelog

October 4, 2024 (version 4.0) In SMAUG-T v4.0, there are some changes in its design and parameters to make the implementation constant time (which had some issues in its previous version) while maintaining the performance. We now avoid using the fixed-weight sampler (HWT) during Encap and Decap. Instead, we changed the distribution of the ephemeral randomness to a non-fixed-weight but sparse ternary vector. This can be viewed as a narrower version of the Centered Binomial Distribution. The parameters are changed accordingly, e.g., the secret key with larger Hamming weights and ephemeral randomness with new sparse distributions. This leads to smaller sizes and smaller DFPs while maintaining security against the recently reported attacks.

The polynomial multiplication is also modified to use the coefficient representation of the secret key instead of using their index to avoid the side-channel attacks and to utilize super-fast AVX optimizations on NTT. We use NTT/Toom-Cook (like Saber).

February 23, 2024 (version 3.0) The two schemes SMAUG and TiGER are merged to SMAUG-T, taking the advantageous features from both schemes. Along with the three SMAUG parameter sets (renamed as SMAUG-T128, 192, 256), a new parameter set TiMER is added, which allows a much lower decryption failure probability, thanks to the error reconciliation from D2 encoding.

A countermeasure was included for the side channel analysis as some vulnerabilities were reported in the KpqC round 1. Hamming weight sampling has been changed and applied to the default, and dGaussian sampling with hiding is provided as an additional implementation.

An optimized implementation with AVX vectorization is also provided, which reports 1.7-1.8x speed-ups. For a fair comparison to other KpqC candidates that provide implementations using so-called 90s symmetric primitives, we also provide an optimized implementation using the 90s, which reports 2.5-3.0x speed-ups compared to the reference implementation.

October 30, 2023 (version 2.0) First, we updated the hamming weight sampler HWT, which was not running at a constant time due to the dependency on the number of hash calls. The new hamming weight sampler is a hybrid of the previous HWT algorithm, which was adopted from SampleInBall algorithm in Dilthium, and the constant weight word sampler [61]. We verified that the new sampler runs at a constant time with a fixed number of hash calls. With the new hamming weight sampler and the partly optimized reference code, SMAUG is now 17% faster than the previous version.

Second, we give an additional security analysis for the choice of the approximate discrete Gaussian sampler. Using the Rényi divergence, it is theoretically guaranteed that the security loss comes from the approximation is minute.

Lastly, we give an additional justification for the decryption failure probability against the state-of-the-art decryption failure attacks, asserting that the

current failure probability of SMAUG is already low enough due to the attack scenarios.

May 23, 2023 (version 1.0) First, we updated the Python script for DFP computation as it was computing the decryption failure probability (DFP) wrongly. Note that the script was missing in the submission file, but included in our website. The parameter sets for NIST's security levels 3 and 5 had higher DFPs than they were reported in the KpqC round 1 submission. As a result, the parameter sets are updated.

Second, we additionally compress the ciphertexts. As compression makes the error larger, we exploit the balance between the sizes and DFP.

Third, we put additional cost estimations on some algebraic and topological attacks: Arora-Ge [7], Coded-BKW [43], and Meet-LWE [57] attacks. We note that the previous parameter sets were all in a secure region against these attacks; however, for the new parameter sets, we aim to have more security margins. We put our code for estimating the cost of the Meet-LWE attack in the Python script.

Based on the above three updates, we changed our recommended parameter sets. As $q = 1024$ is not available anymore for sufficient DFPs in the security levels 3 and 5, we move to $q = 2048$ for those levels, resulting in slightly larger public key and secret key sizes. The ciphertext sizes are decreased by at most 96 bytes.

We also update the reference implementation to have a constant running time with much faster speed. It is uploaded to our website: `kpqc.cryptolab.co.kr/smaug-t`.

1 Introduction

SMAUG-T is an efficient post-quantum Key Encapsulation Mechanism (KEM) whose security is based on the hardness of the lattice problems. SMAUG-T follows the approaches using both Learning-With-Errors (LWE) and Learning-With-Roundings (LWR) variants in recent constructions of post-quantum KEMs such as Lizard [27] and RLizard [55]. Using the two lattice problems, SMAUG-T bases its security on their module variant problems as in Kyber [16] or Saber [35]: the public key does not leak the secret key information by the hardness of Modulue-LWE (MLWE) problem, and the ciphertext protects sharing keys based on the hardness of Modulue-LWR (MLWR) problem.

SMAUG-T consists of the underlying Public Key Encryption (PKE) scheme SMAUG-T.PKE and the KEM scheme SMAUG-T.KEM. SMAUG-T.PKE has IND-Distinguishability under Chosen Message Attack (IND-CPA), which can be converted to SMAUG-T.KEM scheme with INDistinguishability under adaptive Chosen Ciphertext Attack (IND-CCA2), through the Fujisaki-Okamoto (FO) transform.

1.1 Design rationale

The design rationale of SMAUG-T aims is to achieve small ciphertext and public key with low computational cost while maintaining security against various attacks. In more detail, we target the following practicality and security requirements considering real applications:

Practicality:

- Both the public key and ciphertext, especially the latter, which is transmitted more frequently, need to be short in order to minimize communication costs.
- As the key exchange protocol is frequently required on various personal devices, a KEM algorithm with low computational costs is more feasible than a high-cost one.
- A small secret key is desirable in restricted environments such as embedded or IoT devices since managing the secure zone is crucial to prevent physical attacks on secret key storage.

Security:

- Security should be concretely guaranteed concerning the attacks on the underlying assumptions, say lattice attacks.
- The low enough Decryption Failure Probability (DFP) is essential to avoid the attacks boosting the failure and exploiting the decryption failures [31,49].
- As KEMs are widely used in various devices and systems, countermeasures against implementation-specific attacks should also be considered.

MLWE and MLWR. SMAUG-T is constructed on the hardness of MLWE and MLWR problems and follow the key structure of Lizard [27] and Ring-Lizard (RLizard) [55]. Since LWE problem has been a well-studied problem for the last two decades, there are many LWE-based schemes (e.g., FrodoKEM [17]). Ring and module LWE problems (RLWE and MLWE) are variants defined over structured lattices and regarded as hard as LWE. Many schemes base their security on RLWE/MLWE (e.g., NewHope [4], Kyber [16] and Saber [35]) for efficiency reasons. We chose the module structure, which enables us to fine-tune security and efficiency in a much more scalable way, unlike standard and ring versions. Since MLWR problem is regarded as hard as MLWE problem unless we overuse the same secret to generate the samples [15], we chose to use MLWR samples for the encryption. By basing the MLWR, we reduce the ciphertext size by $\log q / \log p$ than MLWE instances so that more efficient encryption and decryption are possible.

Quantum Fujisaki-Okamoto transform. SMAUG-T consists of key encapsulation mechanisms SMAUG-T.KEM, and public key encryption schemes SMAUG-T.PKE. On top of the PKE schemes, we construct the KEM schemes using the FO transform [40, 41]. Line of works on FO transforms in the quantum random

oracle model [14, 47, 50, 59] make it possible to analyze the quantum security, i.e., in the Quantum Random Oracle Model (QROM). In particular, we use the FO transform with implicit rejection and no ciphertext contributions ($\mathsf{FO}^{\not\perp}_m$) following [48].

Sparse secret key and ephemeral secret. We design the key generation algorithm based on MLWE problem using sparse secret. We use sparse ternary polynomials for the secret key and the ephemeral polynomial vectors based on the hardness reduction on the LWE problem using sparse secret [26]. We take advantage of the sparsity, e.g., significantly smaller secret keys. In particular, the small secret makes SMAUG-T more feasible in IoT devices having restricted resources. Specifically, we choose to use a fixed Hamming weight for the secret keys and a non-fixed Hamming weight for the ephemeral secret, a sparse version of the Centered Binomial Distribution (CBD), for secure implementation.

Choice of moduli. All our parameter sets use powers of two moduli. This choice makes SMAUG-T enjoy faster encapsulation using simple bit shiftings, easy uniform samplings, and scalings. The power-of-two moduli makes it hard to apply the Number Theoretic Transform (NTT) on the polynomial multiplications. However, by embedding the power-of-two arithmetic into a larger NTT prime arithmetic, SMAUG-T achieves fast speeds.

Negligible decapsulation failures. Since we base the security on the lattice problems, noise is inherent. Thus decryption result of a SMAUG-T.PKE ciphertext could be different from the original message. We balance the sizes, DFP, and security of SMAUG-T by fine-tuning the parameters while maintaining the DFP to be negligible. In addition, additional parameter set TiMER uses the D2 encoding and error reconciliation as in NewHope [4, 58], to further decrease the DFP and the sizes.

SMAUG-T. We give estimated security and sizes for SMAUG-T parameter sets in Table 1, where the complete version of it can be found in Section 5.2. The sizes are given in bytes, and DFP is given logarithm base two.

The security estimator, the reference codes, and the optimized implementations are available on our website: `www.kpqc.cryptolab.co.kr/smaug-t`.

1.2 Advantages and limitations

Advantages. The security of SMAUG-T relies on the hardness of the lattice problems MLWE and MLWR, which enable balancing between security and efficiency. In terms of sizes, SMAUG-T has smaller ciphertext sizes compared to Kyber or Saber, which is the smallest ciphertext size among the recent practical lattice-based KEMs. In terms of DFP, SMAUG-T achieves low enough DFP, which is less than or similar to that of Saber. SMAUG-T parameter sets do not

Parameters sets	TiMER	SMAUG-T128	SMAUG-T192	SMAUG-T256
Target security	1	1	3	5
(n, k)	$(256, 2)$	$(256, 2)$	$(256, 3)$	$(256, 4)$
q	1024	1024	2048	2048
(p, p')	$(256, 8)$	$(256, 32)$	$(512, 16)$	$(512, 128)$
Classical core-SVP hardness	119.7	119.7	180.2	250.1
Quantum core-SVP hardness	105.4	105.4	158.6	221.0
Decryption failure probability	-161.0	-118.3	-179.2	-194.2
Secret key size	832(160)	832(160)	1312(224)	1792(352)
Public key	672	672	1088	1440
Ciphertext	608	672	992	1376

Table 1: Security and sizes for our parameter sets.

use Error Correction Code (ECC) to avoid possible side-channel attacks, except for the TiMER parameter set. TiMER benefits from the single-bit error correcting D2 encoding, which is masking-friendly from its constructions. Implementation-wise, encapsulation and decapsulation of SMAUG-T can be done efficiently using NTT. Each sub-procedure are masking friendly, against the physical attacks. We give the constant-time C reference code and AVX optimization, which validates the completeness and efficiency of SMAUG-T.

Limitations. We use MLWR problem, which has been studied shorter than MLWE or LWE problems; however, it has a security reduction to MLWE. MLWE problem with a sparse secret has a similar issue but has been studied much longer and is used in various applications, e.g., homomorphic encryptions. As we use MLWE problem for the secret key security, larger public key sizes than Saber are inherent. It can be seen as a trade-off between the public key size versus performance with a smaller secret key size.

2 Preliminaries

2.1 Notation

We denote matrices with bold and upper case letters (e.g., $\mathbf{A}$) and vectors with bold type and lower case letters (e.g., $\mathbf{b}$). Unless otherwise stated, the vector is a column vector.

We define a polynomial ring $\mathcal{R} = \mathbb{Z}[x]/(x^n + 1)$ where n is a power of 2 integers and denote a quotient ring by $\mathcal{R}_q = \mathbb{Z}[x]/(q, x^n + 1) = \mathbb{Z}_q[x]/(x^n + 1)$ for a positive integer q.

For an integer η, we denote the set of polynomials of degree less than n with coefficients in $[-\eta, \eta] \cap \mathbb{Z}$ as S_η. Let $\tilde{S}_\eta$ be a set of polynomials of degree less than n with coefficients in $[-\eta, \eta) \cap \mathbb{Z}$. We denote a uniform distribution over a discrete

set C as $U(C)$. We denote a zero-centered discrete Gaussian distribution with standard deviation σ as $\mathcal{D}_{\mathbb{Z},\sigma}$. We define Rényi divergence of order α between two probability distributions P and Q such that $\mathrm{Supp}(P) \subseteq \mathrm{Supp}(Q)$ as

$$R_\alpha(P\|Q) = \left(\sum_{x \in \mathrm{Supp}(P)} \frac{P(x)^\alpha}{Q(x)^{\alpha-1}} \right)^{1/(\alpha-1)},$$

where $\mathrm{Supp}(D)$ for a distribution D is defined as $\mathrm{Supp}(D) = \{x \in D : D(x) \neq 0\}$. We denote a binomial distribution with a parameter n and a probability p as $B(n,p)$. We denote the Centered Binomial Distribution (CBD) with a parameter d as CBD_d, where the samples range from $-d$ to d.

2.2 Lattice assumptions

We define some well-known lattice assumptions MLWE and MLWR on the structured Euclidean lattices.

Definition 1 (Decision-MLWE$_{n,q,k,\ell,\eta}$). *For positive integers q, k, ℓ, η and the dimension n of $\mathcal{R}$, we say that the advantage of an adversary $\mathcal{A}$ solving the decision-MLWE$_{n,q,k,\ell,\eta}$ problem is*

$$\begin{aligned}\mathsf{Adv}^{MLWE}_{n,q,k,\ell,\eta}(\mathcal{A}) = \big| &\Pr\left[b = 1 \mid \mathbf{A} \leftarrow \mathcal{R}_q^{k\times\ell}; \mathbf{b} \leftarrow \mathcal{R}_q^k; b \leftarrow \mathcal{A}(\mathbf{A}, \mathbf{b})\right] \\ &- \Pr\left[b = 1 \mid \mathbf{A} \leftarrow \mathcal{R}_q^{k\times\ell}; (\mathbf{s}, \mathbf{e}) \leftarrow S_\eta^\ell \times S_\eta^k; b \leftarrow \mathcal{A}(\mathbf{A}, \mathbf{A}\cdot\mathbf{s} + \mathbf{e})\right] \big|\end{aligned}$$

Definition 2 (Decision-MLWR$_{n,p,q,k,\ell,\eta}$). *For positive integers p, q, k, ℓ, η with $q \geq p \geq 2$ and the dimension n of $\mathcal{R}$, we say that the advantage of an adversary $\mathcal{A}$ solving the decision-MLWR$_{n,p,q,k,\ell,\eta}$ problem is*

$$\begin{aligned}\mathsf{Adv}^{MLWR}_{n,p,q,k,\ell,\eta}(\mathcal{A}) = \big| &\Pr\left[b = 1 \mid \mathbf{A} \leftarrow \mathcal{R}_p^{k\times\ell}; \mathbf{b} \leftarrow \mathcal{R}_q^k; b \leftarrow \mathcal{A}(\mathbf{A}, \mathbf{b})\right] \\ &- \Pr\left[b = 1 \mid \mathbf{A} \leftarrow \mathcal{R}_q^{k\times\ell}; \mathbf{s} \leftarrow S_\eta^\ell; b \leftarrow \mathcal{A}(\mathbf{A}, \lfloor p/q \cdot \mathbf{A}\cdot\mathbf{s} \rceil)\right] \big|\end{aligned}$$

2.3 Public key encryption and key encapsulation mechanism

We recap the formalisms of PKE and KEM.

Definition 3 (PKE). *A public key encryption scheme is a tuple of* PPT *algorithms* $\mathsf{PKE} = (\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ *with the following specifications:*

- KeyGen*: a probabilistic algorithm that outputs a public key* pk *and a secret key* sk*;*
- Enc*: a probabilistic algorithm that takes as input a public key* pk *and a message μ and outputs a ciphertext* ct*;*
- Dec*: a deterministic algorithm that takes as input a secret key* sk *and a ciphertext* ct *and outputs a message μ.*

Let $0 < \delta < 1$. We say that it is $(1-\delta)$-correct if for any $(\mathsf{pk}, \mathsf{sk})$ generated from KeyGen *and μ,*

$$Pr[\mathsf{Dec}(\mathsf{sk}, \mathsf{Enc}(\mathsf{pk}, \mu)) \neq \mu] \leq \delta,$$

where the probability is taken over the randomness of the encryption algorithm. We call the above probability decryption failure probability (DFP). In addition, we say that it is correct in the (Q)ROM if the probability is taken over the randomness of the (quantum) random oracle, modeling the hash function.

Definition 4 (KEM). *A key encapsulation mechanism scheme is a tuple of* PPT *algorithms* $\mathsf{KEM} = (\mathsf{KeyGen}, \mathsf{Encap}, \mathsf{Decap})$ *with the following specifications:*

- KeyGen*: a probabilistic algorithm that outputs a public key* pk *and a secret key* sk*;*
- Encap*: a probabilistic algorithm that takes as input a public key* pk *and outputs a sharing key K and a ciphertext* ct*;*
- Decap*: a deterministic algorithm that takes input a secret key* sk *and a ciphertext* ct *and outputs a sharing key K.*

The correctness of KEM *is defined similarly to that of* PKE.

We give the advantage function for a IND-CPA attacker against PKE.

Definition 5 (IND-CPA security of PKE). *For a (quantum) adversary $\mathcal{A}$ against a public key encryption scheme* $\mathsf{PKE} = (\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$*, we define the* IND-CPA *advantage of $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ as follows:*

$$\mathsf{Adv}^{\mathsf{IND\text{-}CPA}}_{\mathsf{PKE}}(\mathcal{A}) = \left| \Pr_{(\mathsf{pk},\mathsf{sk})} \left[b = b' \;\middle|\; \begin{array}{c} (\mu_0, \mu_1, st) \leftarrow \mathcal{A}_1(\mathsf{pk});\ b \leftarrow \{0,1\}; \\ \mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{pk}, \mu_b);\ b' \leftarrow \mathcal{A}_2(\mathsf{pk}, \mathsf{ct}, st) \end{array} \right] - \frac{1}{2} \right|.$$

The probability is taken over the randomness of $\mathcal{A}$ and $(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(1^\lambda)$.

We then define two advantage functions for IND-CPA and IND-CCA2 attackers.

Definition 6 (IND-CPA and IND-CCA security of KEM). *For a (quantum) adversary $\mathcal{A}$ against a key encapsulation mechanism* $\mathsf{KEM} = (\mathsf{KeyGen}, \mathsf{Encap}, \mathsf{Decap})$*, we define the* IND-CPA *advantage of $\mathcal{A}$ as follows:*

$$\mathsf{Adv}^{\mathsf{IND\text{-}CPA}}_{\mathsf{KEM}}(\mathcal{A}) = \left| \Pr_{(\mathsf{pk},\mathsf{sk})} \left[b = b' \;\middle|\; \begin{array}{c} b \leftarrow \{0,1\};\ (K_0, \mathsf{ct}) \leftarrow \mathsf{Encap}(\mathsf{pk}); \\ K_1 \leftarrow \mathcal{K};\ b' \leftarrow \mathcal{A}(\mathsf{pk}, \mathsf{ct}, K_b) \end{array} \right] - \frac{1}{2} \right|.$$

The probability is taken over the randomness of $\mathcal{A}$ and $(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(1^\lambda)$. *The* IND-CCA *advantage of $\mathcal{A}$ is defined similarly except that the adversary can query* $\mathsf{Decap}(\mathsf{sk}, \cdot)$ *oracle on any ciphertext* $\mathsf{ct}' (\neq \mathsf{ct})$.

We can then define the (quantum) security notions of PKE and KEM in the (Q)ROM as follows.

Definition 7 ((Q)ROM security of PKE and KEM). *For $T, \epsilon > 0$, we say that a scheme $\mathcal{S} \in \{\mathsf{PKE}, \mathsf{KEM}\}$ is (T, ϵ)-ATK secure in the (Q)ROM if for any (quantum) adversary $\mathcal{A}$ with runtime $\leq T$ given classical access to $\mathcal{O}$ and (quantum) access to a random oracle H, it holds that $\mathsf{Adv}^{\mathsf{ATK}}_{\mathcal{S}}(\mathcal{A}) < \epsilon$, where*

$$\mathcal{O} = \begin{cases} \mathsf{Enc} & \text{if } \mathcal{S} = \mathsf{PKE} \text{ and } \mathsf{ATK} \in \{\mathsf{OW\text{-}CPA}, \mathsf{IND\text{-}CPA}\}, \\ \mathsf{Encap} & \text{if } \mathcal{S} = \mathsf{KEM} \text{ and } \mathsf{ATK} = \mathsf{IND\text{-}CPA}, \\ \mathsf{Encap}, \mathsf{Decap}(\mathsf{sk}, \cdot) & \text{if } \mathcal{S} = \mathsf{KEM} \text{ and } \mathsf{ATK} = \mathsf{IND\text{-}CCA}. \end{cases}$$

2.4 Fujisaki-Okamoto transform

Fujiskai and Okamoto proposed a novel generic transform [40, 41] that turns a weakly secure PKE scheme into a strongly secure PKE scheme in the Random Oracle Model (ROM), and various variants have been proposed to deal with tightness, non-correct PKEs, and in the quantum setting, i.e., QROM. Here, we recall the FO transformation for KEM as introduced by Dent [33] and revisited by Hofheinz et al. [47], Bindel et al. [13], and Hövelmanns et al. [48].

The original FO transforms $\mathsf{FO}^{\perp}_m$ constructs a KEM from a deterministic PKE, i.e., a de-randomized version. The encapsulation randomly samples a message m and uses the message's hash value $G(m)$ as randomness for encryption, generating a ciphertext. The sharing key $K = H(m)$ is generated by hashing (with different hash functions) the message. In the decapsulation, it first decrypts the ciphertext and recovers the message, m'. If it fails to decrypt, it outputs $\perp$. If the "re-encryption" of the recovered message is not equal to the received ciphertext, it also outputs $\perp$. The sharing key can be generated by hashing the recovered message.

In the quantum setting, however, the FO transform with "implicit rejection" ($\mathsf{FO}^{\not\perp}_m$) has a tighter security proof than the original version, which implicitly outputs a pseudo-random sharing key if the re-encryption fails. We recap the QROM proof of Bindel et al. [13] allowing the KEMs constructed over non-perfect PKEs to have $\mathsf{IND\text{-}CCA}$ security.

Theorem 1 ([13], Theorem 1 & 2). *Let G and H be quantum-accessible random oracles, and the deterministic PKE is ϵ-injective. Then the advantage of $\mathsf{IND\text{-}CCA}$ attacker $\mathcal{A}$ with at most Q_{Dec} decryption queries and Q_G and Q_H hash queries at depth at most d_G and d_H, respectively, is*

$$\mathsf{Adv}^{\mathsf{IND\text{-}CCA}}_{\mathsf{KEM}}(\mathcal{A}) \leq 2\sqrt{(d_G+2)\left(\mathsf{Adv}^{\mathsf{IND\text{-}CPA}}_{\mathsf{PKE}}(\mathcal{B}_1) + 8(Q_G+1)/|\mathcal{M}|\right)} + \mathsf{Adv}^{DF}_{\mathsf{PKE}}(\mathcal{B}_2) + 4\sqrt{d_H Q_H/|\mathcal{M}|} + \epsilon,$$

where $\mathcal{B}_1$ is an $\mathsf{IND\text{-}CPA}$ adversary on PKE and $\mathcal{B}_2$ is an adversary against finding a decryption failing ciphertext, returning at most Q_{Dec} ciphertexts.

3 Design choices

In this section, we explain the design choices for SMAUG-T.

3.1 MLWE public key and MLWR ciphertext

One of the core designs of SMAUG-T uses the MLWE hardness for its secret key security and MLWR hardness for its message security. This choice is adapted from Lizard and RLizard, which use LWE/LWR and RLWE/RLWR, respectively. Using both LWE and LWR variant problems makes the conceptual security distinction between the secret key and the ephemeral sharing key: a more conservative secret key with more efficient en/decapsulations. This can be viewed as a trade-off between "conservative" and "efficient" designs. Combined with the sparse secret, bringing the LWE-based key generation to the LWR-based scheme enables balancing the speed and the DFP.

3.1.1 Public key. Public key of SMAUG-T consists of a vector $\mathbf{b}$ over a polynomial ring $\mathcal{R}_q$ and a matrix $\mathbf{A}$, which can be viewed as an MLWE sample,

$$(\mathbf{A}, \mathbf{b} = -\mathbf{A}^\top \mathbf{s} + \mathbf{e}) \in \mathcal{R}_q^{k \times k} \times \mathcal{R}_q^k,$$

where $\mathbf{s}$ is a ternary secret polynomial with hamming weight h_s, and $\mathbf{e}$ is an error sampled from discrete Gaussian distribution with standard deviation σ. We now specify the uniform matrix sampling algorithm for $\mathbf{A} \in \mathcal{R}_q^{k \times k}$ in Figure 1. It is adapted from the pseudorandom generator gen in Saber [32].

```
expandA(seed):                                          ▷ seed ∈ {0,1}^256
1: buf ← XOF(seed)
2: for i from 0 to k − 1 do
3:     A[i] = bytes_to_Rq(buf + polybytes · i)          ▷ Convert to ring elements
4: return A
```

Fig. 1: Uniform random matrix sampler, expandA.

We note that the public key of SMAUG-T consists of $\mathbf{b}$ and the seed of $\mathbf{A}$.

3.1.2 Ciphertext. The ciphertext of SMAUG-T is a tuple of a vector $\mathbf{c}_1 \in \mathcal{R}_p^k$ and a polynomial $c_2 \in \mathcal{R}_{p'}$. The ciphertext is generated by multiplying a random vector $\mathbf{r}$ to the public key; then it is scaled and rounded as,

$$\mathbf{c} = \begin{bmatrix} \mathbf{c}_1 \\ c_2 \end{bmatrix} = \left\lfloor \frac{p}{q} \cdot \begin{pmatrix} \mathbf{A} \\ \mathbf{b}^\top \end{pmatrix} \cdot \mathbf{r} \right\rceil + \frac{p}{t} \cdot \begin{bmatrix} 0 \\ \mu \end{bmatrix},$$

Along with the public key, it can be treated as an MLWR sample added by a scaled message as $(\mathbf{A}', \lfloor p/q \cdot \mathbf{A}' \cdot \mathbf{r} \rceil) + (0, \mu')$, where $\mathbf{A}'$ is a concatenated matrix of $\mathbf{A}$ and $\mathbf{b}^\top$.

The ciphertext can be further compressed by scaling the second component c_2 by p'/p, resulting in a shorter ciphertext but a larger error. We note that the

public key can be compressed with the same technique. However, it introduces a more significant error, so we do not compress the public key in SMAUG-T.

We call the random vector $\mathbf{r}$ the ephemeral secret, which is a sparse ternary vector. Note that the secret and the ephemeral secret are both sparse ternary vectors; however, we sample them from different distributions using different samplers.

3.2 Sparse secret

We use the sparse ternary distribution for the randomnesses, $\mathbf{s}$ and $\mathbf{r}$. In the following, we will discuss the advantages of the sparse secret and give the sampling algorithm. Notably, we use two different sampling algorithms for the sparse secrets: HWT, a fixed Hamming weight sampler for the secret key $\mathbf{s}$, and spCBD, a non-fixed Hamming weight sampler for ephemeral secret $\mathbf{r}$, respectively.

3.2.1 Advantage of using sparse secret. The sparse secret is widely used in homomorphic encryption to reduce the noise propagation during the homomorphic operations [19,25,44] and to speed up the computations. As the lattice-based KEM schemes have inherent decryption error from LWE or LWR noise, the sparse secret can lower this decryption error and improve the performance of KEMs.

Concretely, the decryption error can be expressed as $\langle \mathbf{e}, \mathbf{r} \rangle + \langle \mathbf{e}_1, \mathbf{s} \rangle + e_2$, where $\mathbf{s}$ is a secret key, $\mathbf{r}$ is a randomness used for encryption, $\mathbf{e} \leftarrow \chi_{pk}^{k}$ is a noise added in public key, and $(\mathbf{e}_1, e_2) \leftarrow \chi_{ct}^{k+1}$ is a noise added in ciphertext. As the vectors $\mathbf{r}$ and $\mathbf{s}$ are ternary, each coefficient of the decryption error is a signed addition of h_r variables from χ_{pk} and $h_s + 1$ variables from χ_{ct}. The magnitude of the decryption error depends greatly on the Hamming weights h_r and h_s; thus, we can take advantage of the sparse secrets.

On the other hand, as the sparse secret reduces the secret key entropy, the hardness of the lattice problem may be decreased. For the security of LWE problem using sparse secret, a series of works have been done, including [26] for asymptotic security based on the reductions to worst-case lattice problems, and [12, 37, 63] for concrete security. Independent of the secret distribution, the module variant (MLWE) is regarded as hard as LWE problem with appropriate parameters, including a smaller modulus. We also exploit the reductions from ordinary MLWE to MLWE using sparse secret or small errors [20]. The MLWR problem also has a simple reduction from MLWE independent of the secret distribution, and its concrete security is heuristically discussed in [32].

Since SMAUG-T uses a sparse secret key $\mathbf{s}$ and a sparse randomness $\mathbf{r}$, the security of SMAUG-T is based on the hardness of MLWE and MLWR problems using sparse secret. For the specific parameters, we exploit the lattice-estimator [1], which covers most of the recent lattice attacks, and also consider some attacks not included in the estimator. Using a smaller modulus, SMAUG-T can maintain high security, as in Kyber or Saber.

3.2.2 Hamming weight sampler Our Hamming weight sampler, HWT_h, is a shuffling-based algorithm that originated from [38], which has no bias on its output and can be realized in a constant-time implementation. This algorithm outperforms other constant-time samplers, such as the sorting-based one or the bounded-rejection-based one. We first describe a subroutine of the shuffling-based sampler in Figure 2, which generates unbiased random integers. An array of integers **si** from an input seed array where $\mathbf{si}_i$ is uniformly sampled from integers $[0, 1, ..., n-i]$ without bias.

REJ_SAMPLE_MOD(rand): ▷ rand is an array of 16-bit integers

1: $j = n$
2: $\mathsf{t} = t_0, t_1, ..., t_{n-1} = 0, 0, ..., 0$
3: **for** i from 0 to $n-1$ **do**
4: $\quad w = 2^L \mod (n-i)$
5: $\quad m = \mathsf{rand}_i \cdot (n-i)$
6: $\quad l = m \mod 2^L$
7: $\quad$ **while** $l < w$ **do**
8: $\quad\quad m = \mathsf{rand}_j \cdot (n-i), \quad j = j+1$
9: $\quad\quad l = m$
10: $\quad \mathsf{t}_i = m \gg L$
11: **return** t

Fig. 2: Algorithm for generating unbiased uniformly random integers

HWT_h(seed):

1: $\mathsf{v} = v_0, v_1, ..., v_{n-1} = 0, 0, ..., 0$
2: buf, sign= PRF(seed)
3: si = REJ_SAMPLE_MOD(buf)
4: $c_0 = n - h$
5: **for** i from 0 to $n-1$ **do** ▷ Binary fixed-weight sampling
6: $\quad t = (\mathsf{si}_i - c_0) \gg 15$
7: $\quad c_0 = c_0 + t_0, \quad \mathsf{v}_i = 1 + t$
8: **for** i from 0 to $n-1$ **do** ▷ Transform to ternary
9: $\quad \mathsf{v}_i = (-\mathsf{v}_i) \wedge ((\mathsf{sign}_i \wedge 0\text{x}02) - 1)$
10: **return** v

Fig. 3: Ternary fixed Hamming weight sampling by shuffling

Then, we introduce the Hamming weight sampler algorithm in Figure 3, which performs a ternary fixed-weight sampling by shuffling, which is slightly modified from [38]. This process generates binary fixed-weight as stated in lines 5 to 11, then transforms to ternary representation using the random bits in sign generated in line 3.

3.2.3 Sparse CBD sampler. Inspired by the efficient CBD sampler from New Hope [4] (and many other KEM schemes) and the approximate discrete Gaussian sampler from SMAUG [24], we introduce a boolean-based efficient sparse ternary sampler, which we call sparse CBD sampler (spCBD).

The spCBD sampler takes input a probability $r < 1/2$ and outputs a signed bit from $\{-1, 0, 1\}$ with a probability mass function $f : \{-1, 0, 1\} \to [0, 1]$ given as

$$f(-1) = f(1) = r,\ f(0) = 1 - 2r\ .$$

This can be naturally extended to a vector of signed bits, where each coordinate follows the same distribution. The resulting vector is a sparse ternary vector. However, as each coordinate is probabilistically sampled, the vector's Hamming weight is not a fixed value. The Hamming follows the binomial distribution as

$$h \sim B(n, 2r)\ ,$$

where h is the Hamming weight and n is the length of the vector.

We remark that, especially when the denominator of the probability r is a power-of-two integer, say 2^k, the spCBD sampler can be efficiently instantiated by sampling $k + 1$ random bits and applying only the boolean operations. We present two spCBD samplers we will use, which randomly sample from the spCBD distribution with the probability parameter $r = 1/8$ and $r = 3/16$ in Figures 4.

$\underline{\mathsf{spCBD}_{1/8}}$:

1: $a, b, c \leftarrow \{0, 1\}$
2: $t \leftarrow a \wedge b$
3: $\mathsf{sign} \leftarrow ((c \ll 1) \wedge \mathsf{0x02}) - 1$
4: **return** $t \cdot \mathsf{sign}$ ▷ Distribution: {−1: 1/8, 0: 3/4, 1: 1/8}

$\underline{\mathsf{spCBD}_{3/16}}$:

1: $a, b, c, d \leftarrow \{0, 1\}$
2: $t \leftarrow a \wedge b \vee c$
3: $\mathsf{sign} \leftarrow ((d \ll 1) \wedge \mathsf{0x02}) - 1$
4: **return** $t \cdot \mathsf{sign}$ ▷ Distribution: {−1: 3/16, 0: 5/8, 1: 3/16}

Fig. 4: Sparse CBD sampler for $r = 1/8$ and $3/16$.

We further note that the distribution $\mathsf{spCBD}_{1/4}$ is equal to CBD with a parameter $d = 1$, i.e., CBD_1, which outputs a ternary secret.

3.3 Discrete Gaussian noise

3.3.1 Using approximate discrete Gaussian noise. Our design choice for the noise distribution in MLWE follows the conventional discrete Gaussian distribution, but with approximated CDTs following the approaches in

FrodoKEM [17]. As a result, we use a discrete Gaussian noise for the public key generation, which is approximated to a narrow distribution. As this approximated discrete Gaussian noise is used only for the public key, we can efficiently bound the security loss from above. Considering the narrow discrete Gaussian noise, we give a theoretical justification based on Rényi divergence to guarantee the security of SMAUG-T.

In SMAUG-T, the narrow discrete Gaussian noise is used only for the public key generation. So, the difference in the noise distribution only affects the distinguishing advantage between the games G_2 and G_3 in the proof of Theorem 4. Then, the bound for the distinguishing advantage can also be expressed as

$$\left(\mathsf{Adv}^{\mathsf{MLWE}}_{n,q,k,k,\mathcal{D}_{\mathbb{Z},\sigma}}(\mathcal{B}_2) \cdot R_\alpha(\mathsf{dGaussian}_\sigma \| \mathcal{D}_{\mathbb{Z},\sigma})^{nk}\right)^{1-1/\alpha},$$

assuming the pseudorandomness of $\mathsf{dGaussian}_\sigma$. This is due to Lemma 5.5 in [3]. We note that the key generation calls dGaussian only nk times and that the public key is generated only once.

The advantage bound for SMAUG-T parameter set (see Section 5.2) can be computed directly using the given formula; for TiMER parameter set (Resp. SMAUG-T128, 192, and 256), the advantage increases by 1.09 (Resp. 1.09, 1.64, and 2.20) bits with $\alpha = 500$. Opposed to the estimated security based on the bound $\mathsf{Adv}^{\mathsf{MLWE}}_{n,q,k,k,\mathsf{dGaussian}_\sigma}(\mathcal{B}_2)$ given in Section 5.2, this new bound provides a more conservative security preventing some possible future attacks that target the noise distribution.

This modification will slightly decrease only the speed of key generation by less than 1.1x.

We also note that the narrow Gaussian noise is already considered when estimating the concrete security (given in Section 5.2) using the explained estimators. The analysis here provides a more conservative security, preventing possible future attacks that target the noise distribution. We also note that in the core-SVP methodology, we only focus on the estimated attack cost of the underlying MLWE and MLWR problems, not based on the security reductions (as done in most of the NIST-submitted schemes) for a fair comparison to Kyber.

3.3.2 dGaussian sampler We construct dGaussian, a constant-time approximate discrete Gaussian noise sampler, upon a Cumulative Distribution Table (CDT) but is not used during sampling, as it is expressed with bit operations.

We first scale the discrete Gaussian distribution and make a CDT approximating the discrete Gaussian distribution. We choose an appropriate scaling factor based on the analysis in [17, 53] using Rényi divergence. We then deploy the Quine-McCluskey method[6] and apply logic minimization technique on the CDT. As a result, even though our dGaussian is constructed upon CDT, it is expressed with bit operations and is constant-time.

[6] We use the python package, from `https://github.com/dreylago/logicmin`.

We describe dGaussian algorithm with $\sigma = 1.0625$ in Figure 5. The algorithm is easily parallelizable and suitable for IoT devices as their memory requirement is low.

$\mathsf{dGaussian}_{1.0625}(x)$:

Require: $x = x_0x_1x_2x_3x_4x_5x_6x_7x_8x_9 \in \{0,1\}^{10}$

1: $s = s_1s_0 = 00 \in \{0,1\}^2$
2: $s_0 = x_0x_1x_2x_3x_4x_5x_7\overline{x_8}$
3: $s_0 \mathrel{+}= (x_0x_3x_4x_5x_6x_8) + (x_1x_3x_4x_5x_6x_8) + (x_2x_3x_4x_5x_6x_8)$
4: $s_0 \mathrel{+}= (\overline{x_2}x_3\overline{x_6}x_8) + (\overline{x_1}x_3\overline{x_6}x_8)$
5: $s_0 \mathrel{+}= (x_6x_7\overline{x_8}) + (\overline{x_5}\overline{x_6}x_8) + (\overline{x_4}\overline{x_6}x_8) + (\overline{x_7}x_8)$
6: $s_1 = (x_1x_2x_4x_5x_7x_8) + (x_3x_4x_5x_7x_8) + (x_6x_7x_8)$
7: $s = (-1)^{x_9} \cdot s$ ▷ $\cdot$ is the arithmetic multiplication
8: **return** s

Fig. 5: Discrete Gaussian sampler with $\sigma = 1.0625$, $\mathsf{dGaussian}_\sigma$.

3.4 Polynomial Multiplication

Despite the sparsity of the secret keys in SMAUG-T, a naive approach to take advantage of the sparsity may expose the scheme to a side-channel attack that exploits the time-variant of executions for polynomial multiplication. Well-known multiplication algorithms that can be implemented with constant-time are NTT and Toom-Cook multiplication.

The moduli of SMAUG-T are all power-of-two integers to efficiently handle rounding by bit shifting and result in non-biased rounding error. To adopt NTT for multiplications in SMAUG-T, the polynomial should be transformed to NTT-friendly ring by switching the modulus. Conversely, the Toom-Cook multiplication is well-suited for handling arbitrary polynomial rings, as its foundation lies in a divide-and-conquer strategy that reduces the problem into smaller subproblems. This approach ultimately relies on classical polynomial multiplication techniques (i.e., schoolbook multiplication) for base cases of sufficiently small size. These multiplications are commonly used in lattice-based PQC schemes, and the performance of these two algorithms varies depending on the degree of the polynomials, the algorithm's parameters, and the operating hardware architecture.

Toom-Cook and Karatsuba. The Toom-Cook [29,64] and the Karatsuba [51] multiplications are efficient algorithms for large integers that split operands and perform multiplications and additions on smaller parts, resulting in lower time complexity. Both achieve sub-quadratic time complexity $O(n^{1+e})$ in the bit-length n where $0 < e < 1$, and can be utilized for the multiplications of polynomials of large degrees.

The Karatsuba multiplication for computing $c(x) = a(x)b(x)$ divides each of degree n polynomials $a(X)$ and $b(x)$ into two sub-polynomials of degree $\frac{n}{2}$. For instance, $a(x)$ is split into $a(x) = a^0(x) + a^1(x)x^{\frac{n}{2}}$, where $a^0(x)$ and $a^1(x)$ are defined as

$$a^0(x) = a_0 + a_1x + a_2x^2 + \cdots + a_{\frac{n}{2}-1}x^{\frac{n}{2}-1}$$
$$a^1(x) = a_{\frac{n}{2}} + a_{\frac{n}{2}+1}x + a_{\frac{n}{2}+2}x^{\frac{n}{2}+2} + \cdots + a_{n-1}x^{\frac{n}{2}-1}.$$

The Karatsuba multiplication computes $c(x)$ with three $\frac{n}{2}$-degree multiplications and some additions rather than 4 polynomial multiplications as follows:

$$\begin{aligned} c(x) = {} & a^0(x)b^0(x) \\ & + \big(\big(a^0(x) + a^1(x)\big)\big(b^0(x) + b^1(x)\big) - \big(a^0(x) + b^0(x)\big)\big(a^1(x) + b^1(x)\big)\big)x^{n/2} \\ & + a^1(x)b^1(x)x^n. \end{aligned}$$

The nested polynomial multiplications can be handled by applying Karatsuba multiplication recursively until the degree of input polynomials are sufficiently small to be executed by the naive multiplication method yielding $\Theta(n^{\log_2 3})$ time complexity by the master theorem for divide-and-conquer recurrences.

Toom-Cook multiplication generalizes Karatsuba multiplication in a way that it splits the degree-n polynomials into k sub-polynomials and handles degree-$\frac{n}{k}$ polynomials in an appropriate manner. It is also possible to compute the multiplications of the sub-polynomials by Karatsuba multiplication. The time complexity of k-way Toom-Cook multiplication is $\Theta(n^{\log_k(2k-1)})$ by the master theorem.

Due to its ability to efficiently handle multiplications on polynomial rings that are not well-suited for NTT, several PQC algorithms, such as those in [36], [45], adopt Toom-Cook multiplication. As in [11], a 256-degree polynomial in SMAUG-T is split into $k = 4$ parts by Toom-Cook, requiring 7 multiplications of 64-degree sub-polynomials. The sub-polynomials are further split by Karatsuba with threshold degree 16. With this choice of k and the threshold degree for Karatsuba, 256-degree polynomial multiplication requires 63 polynomial multiplications for degree 16.

Number Theoretic Transform. The NTT is a widely used method for efficient multiplication in lattice-based cryptography that uses polynomial rings since its quasi-linear complexity $O(n \cdot logn)$. For two polynomials $a(x), b(x) \in \mathcal{R}_q$ the product $a(x) \cdot b(x)$ can be computed as follows where NTT^{-1} denotes the inverse of NTT and $\circ$ is an element-wise multiplication in $\mathbb{Z}_q$.

$$\mathsf{NTT}^{-1}(\mathsf{NTT}(a(x)) \circ \mathsf{NTT}(b(x))).$$

However, NTT has a limitation that requires using an NTT-friendly ring. Specifically, the parameter n should be a power-of-two integer, then the modulus

q must be a prime that is 1 modulo $2n$ to ensure that $\mathbb{Z}_q$ contains primitive n-th or $2n$-th root of unity. Despite its efficiency advantages, some schemes are unable to leverage the NTT due to their use of NTT-unfriendly rings, such as those employing power-of-two modulus or a prime n. Notable examples of such schemes include Saber, NTRU, and SMAUG-T.

On the other hand, it is still possible to use NTT for the polynomial arithmetic in these schemes by embedding the modulus into the NTT prime ring. [28] shows that this approach is more efficient for Saber and NTRU than Toom-Cook in SIMD environments such as AVX2. An efficient AVX2 implementation using this approach is also feasible for SMAUG-T. For the modulus q such that $n \nmid (q-1)$, we represent coefficients within the range $[-\frac{q}{2}, \frac{q}{2})$. Considering that the maximum value of multiplication result coefficient is $n \cdot q^2/4$, if we find an NTT prime Q satisfying $Q > n \cdot q^2/2$ and $n|(Q-1)$, we can use NTT-based multiplication in $\mathcal{R}_Q$ and recover correct result in $\mathcal{R}_q$. If Q becomes too large, it may exceed the 16-bit data type typically used for coefficient. In such cases, by using the Chinese Remainder Theorem (CRT), the product of multiple NTT primes q_i can be used as sufficiently large Q. For SMAUG-T, q_0 = 7,681 and q_1 = 10,753 can be used as NTT primes.

3.5 FO transform, $\mathsf{FO}^{\not\perp}_m$

We construct SMAUG-T upon the FO transform with implicit rejection and without ciphertext contribution to the sharing key generation, say $\mathsf{FO}^{\not\perp}_m$. This choice makes the encapsulation and decapsulation algorithm efficient since the sharing key can be directly generated from a message. The public key is additionally fed into the hash function with the message to avoid multi-target decryption failure attacks. The IND-CCA security of the resulting KEM in the QROM is well-studied in [13, 47, 48].

3.6 D2 encoding

An additional parameter, TiMER, uses D2 encoding. D2 is one of the reconciliation techniques that reduces bandwidth requirements, which was used in NewHope [4]. This technique lowers the decryption failure rate and reduces the ciphertext size by changing the error bound. In Figure 6, we give the description of D2.

To ensure robustness against errors, each bit of the 128-bit message $\mu \in \{0, ..., 255\}^{16}$ is encoded into 2 coefficients by D2Enc. The decoding function D2Dec maps 2 coefficients back to the original key bit. For example, for $n = 256$, take 2 coefficients (each in the range $\{0, ..., q-1\}$), subtract $q/2$ from each of them, accumulate their absolute values, and set the key bit to 0 if the sum is larger than $q/2$ or to 1 otherwise.

$\underline{\mathsf{D2Enc}(\mu \in \{0,\ldots,255\}^{16})}$:

$\mathsf{v} \leftarrow \mathcal{R}_q$
for i from 0 to 15 **do**
  for j from 0 to 7 **do**
    $\mathsf{mask} \leftarrow ((\mu[i] \gg j)\ \&\ 1$
    $\upsilon_{8*i+j+0} \leftarrow \mathsf{mask}\ \&\ (q/2)$
    $\upsilon_{8*i+j+128} \leftarrow \mathsf{mask}\ \&\ (q/2)$
return $\mathsf{v} \in \mathcal{R}_q$

$\underline{\mathsf{D2Dec}(\mathsf{v} \in \mathcal{R}_q)}$:

$\mu \leftarrow \{0,\ldots,255\}^{16}$
for i from 0 to 255 **do**
  $t \leftarrow |(\upsilon_{i+0} \bmod q) - (q-1)/2|$
  $t \leftarrow t + |(\upsilon_{i+128} \bmod q) - (q-1)/2|$
  $t \leftarrow t - q/2$
  $t \leftarrow t \gg 15$
  $\mu[i \gg 3] \leftarrow \mu[i \gg 3] | (t \ll (i\ \&\ 7))$
return $\mu \in \{0,\ldots,255\}^{16}$

Fig. 6: Description of D2 encoding

4 The SMAUG-T

4.1 Specification of SMAUG-T.PKE

We now describe the public key encryption scheme SMAUG-T.PKE in Figure 7 with the following building blocks:

- Pseudo random function PRF for generating $\mathsf{seed}_{\mathbf{A}}$, $\mathsf{seed}_{\mathsf{sk}}$, and $\mathsf{seed}_{\mathbf{e}}$,
- Uniform random matrix sampler expandA for deriving $\mathbf{A}$ from $\mathsf{seed}_{\mathbf{A}}$,
- Discrete Gaussian sampler $\mathsf{dGaussian}_{\sigma}$ for deriving a MLWE noise $\mathbf{e}$ with standard deviation σ from $\mathsf{seed}_{\mathbf{e}}$,
- Hamming weight sampler HWT_h for deriving a sparse ternary $\mathbf{s}$ with Hamming weight $h = h_s$ from $\mathsf{seed}_{\mathsf{sk}}$
- Sparse CBD sampler spCBD_r for deriving a sparse ternary $\mathbf{r}$ with a probability parameter r from $\mathsf{seed}_{\mathbf{r}}$

One of the four parameter sets of SMAUG-T, namely, TiMER, has slightly different features compared to SMAUG-T128 parameter set:

- Reduced message space: from $\{0,1\}^{256}$ to $\{0,1\}^{128}$ for D2 encoding, i.e., $\mu \leftarrow \mathsf{D2Enc}(\mu)$.
- After decryption, the message adjustment process changed from rounding to D2Dec.

The rest of the parts, including the key generations, are done exactly the same as the description in Figure 7.

We then prove the completeness of SMAUG-T.PKE.

$\underline{\mathsf{KeyGen}(1^\lambda)}$:

1: $\mathbf{A} \leftarrow \mathcal{R}_q^{k\times k}$
2: $\mathbf{s} \leftarrow \mathsf{HWT}_{h_s} \in S_\eta^k$
3: $\mathbf{e} \leftarrow \tilde{D}_\sigma \in \mathcal{R}^k$
4: $\mathbf{b} = -\mathbf{A}^\top \cdot \mathbf{s} + \mathbf{e} \in \mathcal{R}_q^k$
5: **return** $\mathsf{pk} = (\mathbf{A}, \mathbf{b})$, $\mathsf{sk} = \mathbf{s}$

$\underline{\mathsf{Enc}(\mathsf{pk}, \mu)}$: ▷ $\mathsf{pk} = (\mathbf{A}, \mathbf{b})$, $\mu \in \mathcal{R}_t$

1: $\mathbf{r} \leftarrow \mathsf{spCBD}_r \in S_\eta^k$
2: $\mathbf{c}_1 = \lfloor p/q \cdot \mathbf{A} \cdot \mathbf{r} \rceil \in \mathcal{R}_p^k$
3: $c_2 = \lfloor p'/q \cdot \langle \mathbf{b}, \mathbf{r} \rangle + p'/t \cdot \mu \rceil \in \mathcal{R}_{p'}$
4: **return** $\mathsf{ct} = (\mathbf{c}_1, c_2)$

$\underline{\mathsf{Dec}(\mathsf{sk}, \mathbf{c})}$: ▷ $\mathsf{sk} = \mathbf{s}$, $\mathbf{c} = (\mathbf{c}_1, c_2)$

1: $\mu' = \lfloor t/p \cdot \langle \mathbf{c}_1, \mathbf{s} \rangle + t/p' \cdot c_2 \rceil \in \mathcal{R}_t$
2: **return** μ'

Fig. 7: Description of SMAUG-T.PKE

Theorem 2 (Completeness of SMAUG-T.PKE). *Let* $\mathbf{A}$, $\mathbf{b}$, $\mathbf{s}$, $\mathbf{e}$, *and* $\mathbf{r}$ *are defined as in Figure 7. Let the moduli* t, p, p', *and* q *satisfy* $t \,|\, p \,|\, q$ *and* $t \,|\, p' \,|\, q$. *Let* $\mathbf{e}_1 \in \mathcal{R}_{\mathbb{Q}}^k$ *and* $e_2 \in \mathcal{R}_{\mathbb{Q}}$ *be the rounding errors introduced from the scalings and roundings of* $\mathbf{A} \cdot \mathbf{r}$ *and* $\mathbf{b}^T \cdot \mathbf{r}$. *That is,* $\mathbf{e}_1 = \frac{q}{p}(\lfloor \frac{p}{q} \cdot \mathbf{A} \cdot \mathbf{r} \rceil \mod p) - (\mathbf{A} \cdot \mathbf{r} \mod q)$ *and* $e_2 = \frac{q}{p'}(\lfloor \frac{p'}{q} \cdot \langle \mathbf{b}, \mathbf{r} \rangle \rceil \mod p') - (\langle \mathbf{b}, \mathbf{r} \rangle \mod q)$. *Let* $\delta = \Pr\left[\|\langle \mathbf{e}, \mathbf{r} \rangle + \langle \mathbf{e}_1, \mathbf{s} \rangle + e_2\|_\infty > \frac{q}{2t}\right]$, *where the probability is taken over the randomness of the encryption. Then* SMAUG-T.PKE *in Figure 7 is* $(1-\delta)$*-correct. That is, for every message* μ *and every key-pair* $(\mathsf{pk}, \mathsf{sk})$ *returned by* $\mathsf{KeyGen}(1^\lambda)$, *the decryption fails with a probability less than* δ.

Proof. By the definition of $\mathbf{e}_1$ and e_2, it holds that $\mathbf{c}_1 = \frac{p}{q} \cdot (\mathbf{A} \cdot \mathbf{r} + \mathbf{e}_1) \mod p$ and $c_2 = \frac{p'}{q} \cdot (\langle \mathbf{b}, \mathbf{r} \rangle + e_2) + \frac{p'}{t} \cdot \mu \mod p'$, where the coefficients of $\mathbf{e}_1$ and e_2 are in $\mathbb{Z} \cap (-\frac{q}{2p}, \frac{q}{2p}]$ and $\mathbb{Z} \cap (-\frac{q}{2p'}, \frac{q}{2p'}]$, respectively. Thus, the decryption of the ciphertext $(\mathbf{c}_1, c_2)$ can be written as

$$\begin{aligned}
\left\lfloor \frac{t}{p} \cdot \langle \mathbf{c}_1, \mathbf{s} \rangle + \frac{t}{p'} \cdot c_2 \right\rceil \bmod t &= \left\lfloor \frac{t}{q} \left(\langle \mathbf{A} \cdot \mathbf{r}, \mathbf{s} \rangle + \langle \mathbf{e}_1, \mathbf{s} \rangle + \langle \mathbf{b}, \mathbf{r} \rangle + e_2 \right) + \mu \right\rceil \bmod t \\
&= \left\lfloor \frac{t}{q} \left(\langle \mathbf{A}^\top \cdot \mathbf{s} + \mathbf{b}, \mathbf{r} \rangle + \langle \mathbf{e}_1, \mathbf{s} \rangle + e_2 \right) + \mu \right\rceil \bmod t \\
&= \mu + \left\lfloor \frac{t}{q} \left(\langle \mathbf{e}, \mathbf{r} \rangle + \langle \mathbf{e}_1, \mathbf{s} \rangle + e_2 \right) \right\rceil \bmod t.
\end{aligned}$$

This is equal to μ if and only if every coefficient of $\langle \mathbf{e}, \mathbf{r} \rangle + \langle \mathbf{e}_1, \mathbf{s} \rangle + e_2$ is in the interval $[-\frac{q}{2t}, \frac{q}{2t})$. It concludes the proof. □

Note, it can be trivially proven that the use of D2 encoding in TiMER parameter set does not change the completeness of SMAUG-T, since the D2 encoding

output can be seen as the message μ in the above proof. The only assumption we require is the completeness of D2 encoding.

4.2 Specification of SMAUG-T.KEM

We introduce the key encapsulation mechanism SMAUG-T.KEM in Figure 8. SMAUG-T.KEM is designed following the Fujisaki-Okamoto transform with implicit rejection using the non-perfectly correct public key encryption SMAUG-T.PKE. The construction of SMAUG-T.KEM involves the use of the following symmetric primitives:

- Hash function H for hashing a public key,
- Hash function G for deriving a sharing key and a seed.

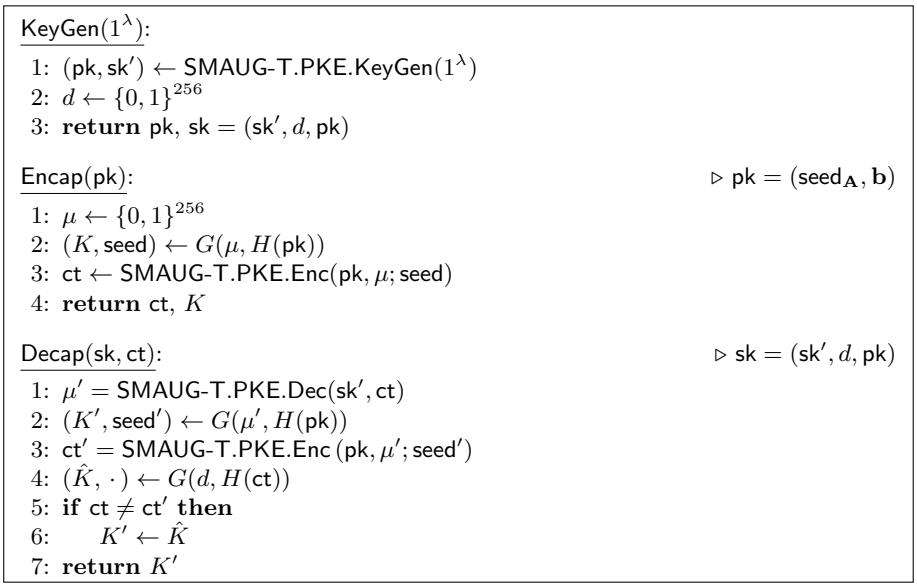

Fig. 8: Description of SMAUG-T.KEM

As in the SMAUG-T.PKE, we can easily construct the TiMER parameter set, which uses the TiMER parameter set of SMAUG-T.PKE in a black-box manner, with the following change:

- Reduced randomness space and entropy for μ, from $\{0,1\}^{256}$ to $\{0,1\}^{128}$

The Fujisaki-Okamoto transform used in Figure 8 defers from the $\mathsf{FO}^{\not\perp}_m$ transform in [48] in encapsulation and decapsulation. When generating the sharing key and randomness, SMAUG-T's Encap utilizes the hashed public key, which

prevents certain multi-target attacks. As for Decap, if $\mathsf{ct} \neq \mathsf{ct}'$ holds, an alternative sharing key should be re-generated so as not to leak failure information against Side-Channel Attacks (SCA). However, even when the failure information is leaked, security can still rely on the explicit FO transform $\mathsf{FO}_m^{\perp}$, recently treated in [47] with a competitive bound.

We also remark that the randomly chosen message μ should be hashed in the environments using a non-cryptographic Random Number Generator (RNG) system. A True Random Number Generator (TRNG) is recommended to sample the message μ in such devices.

We now show the completeness of SMAUG-T.KEM based on the completeness of the underlying public key encryption scheme, SMAUG-T.PKE.

Theorem 3 (Completeness of SMAUG-T.KEM). *We borrow the notations and assumptions from Theorem 2 and Figure 8. Then* SMAUG-T.KEM *in Figure 8 is also* $(1-\delta)$*-correct. That is, for every key-pair* $(\mathsf{pk}, \mathsf{sk})$ *generated by* $\mathsf{KeyGen}(1^\lambda)$*, the shared keys* K *and* K' *are identical with probability larger than* $1-\delta$*.*

Proof. The shared keys K and K' are identical if the decryption succeeds. Assuming the pseudorandomness of the hash function G, the probability of being $K \neq K'$ can be bounded by the DFP of SMAUG-T.PKE. The completeness of SMAUG-T.PKE (Theorem 2) concludes the proof. □

4.3 Security proof

When proving the security of the KEMs constructed using FO transform in the (Q)ROM, on typically relies on the generic reductions from one-wayness or IND-CPA security of the underlying PKE. In the ROM, SMAUG-T.KEM has a tight reduction from the IND-CPA security of the underlying PKE, SMAUG-T.PKE. However, like other lattice-based constructions, the underlying PKE has a chance of decryption failures, which makes the generic reduction unapplicable [59] or non-tight [13,47,48] in the QROM. Therefore, we prove the IND-CCA security of SMAUG-T.KEM based on the non-tight QROM reduction of [13] as explained in Section 2 by proving the IND-CPA security of SMAUG-T.PKE.

Theorem 4 (IND-CPA security of SMAUG-T.PKE). *Assuming pseudorandomness of the underlying sampling algorithms, the* IND-CPA *security of* SMAUG-T. PKE *can be tightly reduced to the decisional MLWE and MLWR problems. Specifically, for any* IND-CPA*-adversary* $\mathcal{A}$ *of* SMAUG-T.PKE*, there exist adversaries* $\mathcal{B}_0$*,* $\mathcal{B}_1$*,* $\mathcal{B}_2$*, and* $\mathcal{B}_3$ *attacking the pseudorandomness of* XOF*, and the pseudorandomness of sampling algorithms, the hardness of MLWE, and the hardness of MLWR, respectively, such that,*

$$\begin{aligned}\mathsf{Adv}^{\mathsf{IND\text{-}CPA}}_{\mathsf{SMAUG\text{-}T.PKE}}(\mathcal{A}) \leq\ & \mathsf{Adv}^{\mathsf{PR}}_{\mathsf{XOF}}(\mathcal{B}_0) + \mathsf{Adv}^{\mathsf{PR}}_{\mathsf{expandA,HWT,dGaussian}}(\mathcal{B}_1) \\ & + \mathsf{Adv}^{\mathsf{MLWE}}_{n,q,k,k}(\mathcal{B}_2) + \mathsf{Adv}^{\mathsf{MLWR}}_{n,p,q,k+1,k}(\mathcal{B}_3).\end{aligned}$$

The secret distribution terms omitted in the last two advantages (of $\mathcal{B}_1$ and $\mathcal{B}_2$) are uniform over ternary polynomials with Hamming weights h_s and h_r, respectively. The error distribution term omitted in the advantage of $\mathcal{B}_2$ is a pseudorandom distribution following the corresponding CDT.

Proof. The proof proceeds by a sequence of hybrid games from G_0 to G_4 defined as follows:

- G_0: the genuine IND-CPA game,
- G_1: identical to G_0, except that the public key is changed into $(\mathbf{A}, \mathbf{b})$,
- G_2: identical to G_1, except that the sampling algorithms are changed into truly random samplings,
- G_3: identical to G_2, except that $\mathbf{b}$ is randomly chosen from $\mathcal{R}_q^k$,
- G_4: identical to G_3, except that the ciphertext is randomly choosen from $\mathcal{R}_p^k \times \mathcal{R}_{p'}$. As a result, the public key and the ciphertexts are truly random.

We denote the advantage of the adversary on each game G_i as Adv_i, where $\mathsf{Adv}_0 = \mathsf{Adv}^{\mathsf{IND\text{-}CPA}}_{\mathsf{SMAUG\text{-}T.PKE}}(\mathcal{A})$ and $\mathsf{Adv}_4 = 0$. Then, it holds that

$$|\mathsf{Adv}_0 - \mathsf{Adv}_1| \leq \mathsf{Adv}^{\mathsf{PR}}_{\mathsf{XOF}}(\mathcal{B}_0),$$

for some adversary $\mathcal{B}_0$ against the pseudorandomness of the extendable output function. Given that the only difference between the transcripts viewed in hybrid games G_1 and G_2 is the randomness sampling, it can be concluded that

$$|\mathsf{Adv}_1 - \mathsf{Adv}_2| \leq \mathsf{Adv}^{\mathsf{PR}}_{\mathsf{expandA,HWT,dGaussian}}(\mathcal{B}_1),$$

for some adversary, $\mathcal{B}_1$ attacking the pseudorandomness of at least one of the samplers. The difference in the games G_2 and G_3 is in the way the polynomial vector $\mathbf{b}$ is sampled. In G_2, it is sampled as part of an MLWE sample, whereas in G_3, it is randomly selected. Thus, the difference in the advantages Adv_2 and Adv_3 can be bounded by $\mathsf{Adv}^{\mathsf{MLWE}}_{n,q,k,k}(\mathcal{B}_2)$, where $\mathcal{B}_2$ is an adversary distinguishing the MLWE samples from random. In the hybrids G_3 and G_4, the only difference is in the way the ciphertexts are generated; they are either randomly chosen from $\mathcal{R}_p^k \times \mathcal{R}_{p'}$ or generated to be $(\mathbf{c}_1, \lfloor p'/p \cdot c_2 \rceil)$, where

$$\begin{bmatrix} \mathbf{c}_1 \\ c_2 \end{bmatrix} = \left\lfloor \frac{p}{q} \cdot \begin{pmatrix} \mathbf{A} \\ \mathbf{b}^\top \end{pmatrix} \cdot \mathbf{r} \right\rceil + \frac{p}{t} \cdot \begin{bmatrix} 0 \\ \mu \end{bmatrix}.$$

If an adversary $\mathcal{A}$ can distinguish the two ciphertexts, we can construct an adversary $\mathcal{B}_3$ distinguishing the MLWR sample from random: *for given a sample* $(\mathbf{A}, \mathbf{b}) \in \mathcal{R}_q^{(k+1)\times k} \times \mathcal{R}_p^{k+1}$, $\mathcal{B}_3$ *rewrites* $\mathbf{b}$ *as* $(\mathbf{b}_1, b_2) \in \mathcal{R}_p^k \times \mathcal{R}_p$, *computes* $(\mathbf{b}_1, \lfloor p'/p \cdot b_2 \rceil)$, *and use* $\mathcal{A}$ *to decide the ciphertext type. The output of* $\mathcal{A}$ *will be the output of* $\mathcal{B}_3$. Therefore, we can conclude the proof by observing that

$$|\mathsf{Adv}_3 - \mathsf{Adv}_4| \leq \mathsf{Adv}^{\mathsf{MLWR}}_{n,p,q,k+1,k}(\mathcal{B}_3).$$

□

Again, the D2 encoding does not introduce any changes in the above proof, as the encoded messages are added to a full random MLWR instances, assuming the MLWR hardness.

The classical IND-CCA security of SMAUG-T.KEM is then obtained directly from FO transforms [47] in the classical random oracle model. Theorem 1 implies the quantum IND-CCA security of SMAUG-T.KEM in the quantum random oracle model.

The TiMER parameter set is well-suited for lightweight IoT environments thanks to its smaller ciphertext size. However, the use of D2 encoding and the smaller randomness space may affect security in the future. For better-ensuring security when using TiMER parameter set, it is recommended to limit the number of Encap/Decap by considering the operating environment.

5 Parameter selection and concrete security

In this section, we first give a concrete security analysis of SMAUG-T and provide the parameter sets.

5.1 Concrete security estimation

We exploit the best-known lattice attacks to estimate the concrete security of SMAUG-T.

5.1.1 Core-SVP methodology. Most of the known attacks are essentially finding a nonzero short vector in Euclidean lattices, using the Block–Korkine–Zolotarev (BKZ) lattice reduction algorithm [22, 46, 60]. BKZ has been used in various lattice-based schemes [2, 16, 34, 39, 65]. The security of the schemes is generally estimated as the time complexity of BKZ in core-SVP hardness introduced in [4]. It depends on the block size β of BKZ reporting the best performance. According to Becker et al. [9] and Chailloux et al. [21], the β-BKZ algorithm takes approximately $2^{0.292\beta+o(\beta)}$ and $2^{0.257\beta+o(\beta)}$ time in the classical and quantum setting, respectively. The polynomial factors and $o(\beta)$ terms in the exponent are ignored. We use the lattice estimator [1] to estimate the concrete security of SMAUG-T in core-SVP hardness.

5.1.2 Beyond Core-SVP methodology. In addition to lattice reduction attacks, we also take into consideration the cost of other types of attacks, e.g., algebraic attacks like the Arora-Ge attack or Coded-BKW attacks, and their variants. In general, these attacks have considerably higher costs and memory requirements compared to previously introduced attacks.

MLWE with fixed Hamming weight secret. We also focus on the attacks not considered in the lattice estimator, specifically those that target sparse secret,

such as Meet-LWE [57] attack. This attack is inspired by Odlyzko's Meet-in-the-Middle approach and involves using representations of ternary secrets in additive shares. The asymptotic attack complexity is claimed as $\mathcal{S}^{0.25}$; however, it is far from the estimated attack costs in SMAUG-T parameter sets. Even the estimated cost has a significant gap with the real attack, due to the hidden costs behind the estimation.

MLWE with spCBD. When using spCBD, the number of non-zero coefficients is not fixed. Attacks like May's Meet-LWE [57] or Lee et al. [54] cannot be directly applied. As the distribution of h follows the binomial distribution centered at $n \cdot 2r$, an attacker can guess $h = n \cdot 2r$ or a value close to it and apply the attack. The probability of a correct guess is

$$\binom{n}{h} \cdot (2r)^h \cdot (1-2r)^{n-h} \ ,$$

which should be considered for the attack cost estimation. We note the value achieves the maximum when $h = \lfloor n \cdot 2r \rfloor$. Therefore, one can estimate the total cost of MLWE with spCBD secret as

$$\min_h \left\{ \frac{\mathsf{ATK}_h}{\binom{n}{h} \cdot (2r)^h \cdot (1-2r)^{n-h}} \right\} \ , \tag{1}$$

where ATK_h is the attack cost of [54] for MLWE with a secret having a fixed Hamming weight of h. For a rough estimation, we follow [54] and assume the attack cost ATK_h is greater than $\binom{n}{h}^{0.21}$, the secret space size to the power of 0.21, resulting in an asymptotic lower bound of the attack cost of

$$\min_h \left\{ \frac{1}{\binom{n}{h}^{0.79} \cdot (2r)^h \cdot (1-2r)^{n-h}} \right\} \ . \tag{2}$$

Depending on the parameters, the use of spCBD increases the attack cost compared to a fixed Hamming weight of $h = n \cdot 2r$ but also decreases the DFP in practice.

We summarize the costs of the algebraic and combinatorial attacks in Table 2. Attack costs for Arora-Ge and Coded-BKW are estimated with lattice estimator [1]. The estimated cost of Arora-Ge attack on SMAUG-T256 is not determined by lattice-estimator, outputting ∞, which is at least a thousand bits of security. The costs for the Meet-LWE attack are estimated with a python script[7] based on May's analysis [57], best among Rep-1 and Rep-2. In addition, we also consider the attack of Lee et al. [54] and its variant.[8] The hardness of an MLWE

[7] The script can be found on the team SMAUG-T website: `http://kpqc.cryptolab.co.kr/`

[8] We remark that the attack of Lee et al. and its cost estimation is not yet verified; however, it is worth adding such a countermeasure to the scheme against the attacks.

sample with the secret of a fixed Hamming weight is given based on the analysis of [54]. For the hardness of an MLWR sample with the secret of non-fixed weight spCBD sampler, we applied a variant of the attack as described in Section 5.1.2: We first find h that minimizes the Equation 2 ($h = 102, 102, 384, 352$) then we calculate the attack cost based on the Equation 1. However, as we are unaware of Lee et al.'s attack cost estimator exactly for each h, we give a lower bound of our attack based on their analysis. By using the $\mathsf{ATK}_{h'}$ that we know ($h' = 100, 100, 264, 348$), satisfying $h' < h$ so that $\mathsf{ATK}_h > \mathsf{ATK}_{h'}$ holds. This means that we only give a lower bound of the attack cost estimation, which can be improved using the estimator of Lee et al.

Parameters sets Security level		TiMER 1	SMAUG-T128 1	SMAUG-T192 3	SMAUG-T256 5
Classical core-SVP		119.7	119.7	180.2	250.1
Algebraic & Combinatorial attacks					
Arora-Ge	time (mem)	693.6 (553.0)	693.6 (553.0)	- (908.9)	- -
BKW	time (mem)	144.7 (132.7)	144.7 (132.7)	213.7 (202.1)	269.0 (257.0)
Meet-LWE	time (mem)	177.2 (157.4)	177.2 (157.4)	295.6 (259.1)	401.4 (353.1)
Lee et al. [54]*	time	(148, 132)	(148, 132)	(236, 241)	(309, 317)

Table 2: Attack costs beyond Core-SVP. The estimated cost of the Arora-Ge attack sometimes overflowed, implying that it requires at least 2^{1000} of operations. For the attack of Lee et al., we apply our modifications detailed in Sections 5.1.2, where the estimated costs are given for both keys (MLWE) and ciphertexts (MLWR).

5.1.3 MLWE hardness. We estimated the cost of the best-known attacks for MLWE, including *primal attack*, *dual attack*, and their hybrid variations, in the core-SVP hardness. We remark that any $\mathsf{MLWE}_{n,q,k,\ell,\eta}$ instance can be viewed as an $\mathsf{LWE}_{q,nk,n\ell,\eta}$ instance. Although the MLWE problem has an additional algebraic structure compared to the LWE problem, no attacks currently take advantage of this structure. Therefore, we assess the hardness of the MLWE problem based on the hardness of the corresponding LWE problem. We also consider the distributions of secret and noise when estimating the concrete security of SMAUG-T. We have also analyzed the costs of recent attacks that aim to target the MLWE problem with sparse secrets. Our narrow discrete Gaussian sampler's tail bound is considered in estimating the security using the lattice estimator.

5.1.4 MLWR hardness. To measure the hardness of the MLWR problem, we treat it as an MLWE problem since no known attack utilizes the deterministic error term in the MLWR structure. Banerjee et al. [8] provided the reduction from the MLWE problem to the MLWR problem, which was subsequently improved in [5,6,15]. Basically, for given an MLWR sample $(\mathbf{A}, \lfloor p/q \cdot \mathbf{A} \cdot \mathbf{s} \rceil \mod p)$ with uniformly chosen $\mathbf{A} \leftarrow \mathcal{R}_q^k$ and $\mathbf{s} \leftarrow \mathcal{R}_p^\ell$, it can be expressed as $(\mathbf{A}, p/q \cdot (\mathbf{A} \cdot \mathbf{s} \mod q) + \mathbf{e} \mod p)$. The MLWR sample can be converted to an MLWE sample over $\mathcal{R}_q$ by multiplying q/p as $(\mathbf{A}, \mathbf{b} = \mathbf{A} \cdot \mathbf{s} + q/p \cdot \mathbf{e} \mod q)$. Assuming that the error term in the resulting MLWE sample is a random variable, uniformly distributed within the interval $(-q/2p, q/2p]$, we can estimate the hardness of the MLWR problem as the hardness of the corresponding MLWE problem.

5.2 Parameter sets

The SMAUG-T is parameterized by various integers such as n, k, q, p, p', t, h_s and h_r, as well as a standard deviation $\sigma > 0$ for the discrete Gaussian noise. Our main focus when selecting these parameters is to minimize the ciphertext size while maintaining security. We first set our ring dimension to $n = 256$ and plaintext modulus to $t = 2$ to have a 256-bit (for SMAUG-T128, 192, 256) or 128-bit (for TiMER) message space. The sharing-key space is 256-bit for all the parameter sets. Then, we search for parameters with enough security to offer the smallest ciphertext size. Starting from parameters with a tiny ciphertext size, we increase the ciphertext size, h_s, r, and σ, then search for the parameters with enough security. Once we have a candidate, we compute the DFP. If it is low enough, we can choose the compression parameter p', but if not, we continue searching for appropriate parameters. If the DFP is low enough, the compression factor p' can be set to a smaller integer.

Table 3 outlines the whole set of recommended parameters corresponding to NIST's security levels 1, 3, and 5. For security levels 3 and 5, we can not find the parameters with $q = 1024$, so we use $q = 2048$. Especially, the standard deviation $\sigma = 1.0625$ is the same across the whole parameter sets.

TiMER, an additional parameter set, further investigates the room for efficiency, introducing the D2 encoding to SMAUG-T128. It has a 64-byte smaller ciphertext size than SMAUG-T128. TiMER sufficiently lowers DFP through D2 encoding and error reconciliation techniques. Thanks to this lowered DFP, p' was reduced from 32 to 8, further compressing the ciphertext.

The core-SVP hardness is estimated via the lattice estimator [1] using the cost model "ADPS16" introduced in [4] and "MATZOV" [56]. In the table, the smaller cost is reported. We assumed that the number of 1s is equal to the number of -1s for simplicity, which conservatively underestimates security.

The security beyond core-SVP is estimated via the lattice estimator [1] and the Python script implementing the Meet-LWE attack cost estimation. It shows the lowest attack costs among coded-BKW, Arora-Ge, and Meet-LWE attack and their variants.

Parameters sets	TiMER	SMAUG-T128	SMAUG-T192	SMAUG-T256
Security level	1	1	3	5
n	256	256	256	256
k	2	2	3	4
(q, p, t)	$(1024, 256, 2)$	$(1024, 256, 2)$	$(2048, 512, 2)$	$(2048, 512, 2)$
p' (compression)	8	32	16	128
h_s (HWT for $\mathbf{s}$)	140	140	264	348
r (spCBD for $\mathbf{r}$)	1/8	1/8	1/4	3/16
σ ($\bar{D}_\sigma$ for errors)	1.0625	1.0625	1.0625	1.0625
Classical core-SVP	119.7	119.7	180.2	250.1
Quantum core-SVP	105.4	105.4	158.6	221.0
Beyond core-SVP	132	132	214	269
DFP	-161.0	-118.3	-179.2	-194.2
Public key	672	672	1088	1440
Ciphertext	608	672	992	1376

Table 3: Parameters for SMAUG-T. Classical and quantum security is given in core-SVP hardness. The DFP (in $\log_2$) and sizes (in bytes) are also given in advance.

5.3 Decryption failure probability

Our primary goal is to push the efficiency of the lattice-based KEMs to the limit while maintaining roughly the same level of security, so we follow the frameworks given in Kyber and Saber. We set the DFPs as small as $\approx 2^{-\lambda}$ for a desired security parameter λ, except for the SMAUG-T256 parameter set. We set the DFP of SMAUG-T256 at least much smaller than that of Kyber and Saber.

The impact of DFP on the security of KEM is still being investigated. However, we can justify why our choice is sufficient for real-world scenarios, focusing on SMAUG-T256. To do so, we make the following assumptions:

1. Each key pair has a limit of $Q_{\text{limit}} = 2^{64}$ decryption queries, as specified in NIST's proposal call.
2. There are approximately 2^{33} people worldwide, each with hundreds of devices. Each device has thousands of *usable* public keys broadcasted for KEM.
3. We introduce an *observable probability* and assume it is far less than 2^{-20}. Even though the decryption failure occurs, it can only be used for an attack when observed. Attackers can observe it through a side-channel attack, which enables the observation of decapsulation failures in the mounted device or through direct communications after key derivation. This allows the detection of decryption failures with a communication per key pair. We assume the two cases can occur much less than 2^{-20}, as they require physically mounted devices or communications with shared keys.

Based on these assumptions, we can deduce that the number of observable decryption failures can be upper bounded by $2^{64+33+10+12} \cdot 2^{-20} = 2^{99}$. Based on the best-known (multi-target) attacks for Saber [30, Figure 7a], the quantum

cost for finding a single failing ciphertext that may lead to a successful attack of SMAUG-T256 is expected to be much higher than 2^{300}, as desired[9]. Regardless of the attack cost estimated above, the scenario of checking the failures in more than 2^{40} different devices is already way too far from the real-world attack scenario.

6 Implementation

In this section, we consider the implementation of SMAUG-T and present the performance for each parameter set. We provide a few C implementations: The constant-time reference implementation of SMAUG-T parameter sets can be found in the `reference_implementation`, and an optimized implementation utilizing AVX2 intrinsics on Intel(R) is included in the `optimized_implementation`. Additionally, the TiMER parameter set, designed for lightweight environments, is available in the `additional_implementation`. Our implementations, along with the supporting scripts, are accessible on our website: `www.kpqc.cryptolab.co.kr/smaug-t`.

6.1 Implementation considerations

The most critically time-consuming component in SMAUG-T is the symmetric primitive. We chose SHA3 as the symmetric variant, which occupies about 30% to 40% of the cycles according to the reference implementation. Being based on the Keccak permutation, SHA3 is not the fastest algorithm in software. Thus its usage may impose performance constraints compared to 90s symmetrics (AES, SHA2). However, similar to how ARM provides hardware acceleration for SHA3 on ARMv8 processors, this is not expected to be a problem in the future.

To measure the achieved performance of SMAUG-T, we also provide an optimized implementation that uses 90s symmetrics. This implementation serves as benchmark code to demonstrate optimized performance by utilizing AES and SHA2 instead of other symmetric algorithms. Consequently, this leads to differences in the test vector when compared to the reference implementation. The optimized implementation using 90s symmetric can be found at `optimized_implementation/kem-90s`.

6.2 Performance

In the reference implementation and additional implementation, we instantiate the hash functions G, H, the extendable output function XOF, and the pseudorandom function PRF with the following symmetric primitives: G and PRF are instantiated with SHAKE256, H is instantiated with SHA3-256, XOF is instantiated with SHAKE128.

[9] Specifically, the number of observable failures must be larger than $1/\beta$ in [30] to observe at least one failing ciphertext. That is, β should be smaller than 2^{-93}. When this is assumed, the quantum cost is then $1/\beta\sqrt{\alpha}$, given in the x-axis.

Table 4 presents the performance results of SMAUG-T. For a fair comparison, we also performed measurements on the same system with identical settings of the reference implementation of Kyber [10]. All benchmarks are obtained on one core of an Intel(R) Core(TM) i7-10700K CPU processor with a clock speed of 3.80GHz. The benchmarking machine has 64 GB of RAM and runs Debian GNU/Linux with Linux kernel version 5.4.0. The implementation is compiled with gcc version 11.4.0, and the compiler flags as indicated in the Makefile included in the submission package.

	Cycles (ref)						**Cycles (AVX2)**					
Schemes	KeyGen		Encap		Decap		KeyGen		Encap		Decap	
TiMER	110	1	100	1	135	1	-		-		-	
SMAUG-T128	110	1	100	1	136	1	38	1	23	1	35	1
Kyber512	128	1.2	158	1.6	187	1.4	27	0.7	39	1.7	29	0.8
SMAUG-T192	219	1	204	1	253	1	57	1	46	1	61	1
Kyber768	209	1	255	1.3	286	1.1	44	0.8	65	1.4	44	0.7
SMAUG-T256	357	1	334	1	414	1	77	1	65	1	86	1
Kyber1024	321	0.9	369	1.1	414	1	60	0.8	79	1.2	63	0.7

Table 4: Median kilocycle counts of 1000 executions for SMAUG-T and Kyber (and their ratios). "ref" refers to the C implementation, while "AVX2" refers to the implementation with AVX2 intrinsics.

In the optimized implementation, we offer two options for the symmetric primitives. The default implementation located in `optimized_implementation/kem` uses all symmetric primitives identically to the reference implementation. In the 90s symmetric implemenation found in `optimized_implementation/kem-90s`, we instantiate the hash functions G, H, the extendable output function XOF, and the pseudo random function PRF with the following symmetric primitives: G is instantiated with SHA2-512, H is instantiated with SHA2-256, and both XOF and PRF are instantiated with AES.

SMAUG-T optimized implementation using AVX2 intrinsics achieved a speed up of about x3 - x5.2, while the optimized implementation using 90s symmetric achieved a speed up of about x4.2 - x7.9. All measurement methods and conditions are identical to those of Table 4.

[10] From `github.com/pq-crystals/kyber` (`518de24`)

7 Side Channel Analysis

SMAUG-T is a scheme based on MLWE and MLWR that has many similarities to Kyber and Saber. As a result of the NIST competition, much research has been conducted on side-channel analysis and countermeasures for Kyber and Saber [10, 18]. These previous findings can also be applied to SMAUG-T. Therefore, we decided to focus our analysis on the characteristic designs in which SMAUG-T differs from Kyber or Saber. While KpqC round 1 focused on timing attacks, power/EM-based attacks are becoming increasingly critical with advanced attack techniques and tools, necessitating proactive countermeasures. In particular, the recently announced clustering attack [62] has become a more lethal threat due to the small number of traces and advances in deep learning technology. Thus, we discuss the security of SMAUG-T against physical attacks based on power/EM.

7.1 Timing analysis

Samplers. At present, SMAUG-T has been carefully implemented to avoid time variations such as branches with respect to secrets. For key generation, a shuffling-based constant-time and unbiased fixed-weight sampler has been used as the fixed-weight sampler. Furthermore, for ephemeral randomness in encapsulation/decapsulation, we propose a new constant-time sparse CBD sampler. This sampler is constructed solely from bit operations, making it highly secure and efficient for implementation. Previous PQC algorithms utilizing Gaussian noises have employed various Gaussian samplers. However, designing Gaussian samplers that operate in constant-time is challenging, and BLISS has suffered from timing attacks [42]. We adopted $\mathsf{dGaussian}_\sigma$, a constant-time implementation well-known for its efficacy, into SMAUG-T to mitigate timing attacks.

D2 encoding and error reconciliation. As mentioned earlier, D2 encoding and error reconciliation were used in NewHope, and due to modulus reduction, the D2 implementation was not constant-time. In NewHope, they solved this problem with constant-time Barrett reduction. On the other hand, in TiMER, since the modulus is all powers of 2, the modulus reduction can be replaced by a shift operation, eliminating the attack surface.

7.2 Differential analysis

$\mathsf{dGaussian}_\sigma$ sampler. Potential vulnerabilitie related to Power/EM-based SCA for $\mathsf{dGaussian}_\sigma$ was reported during KpqC round 1. There were no specific attack scenario and applying this vulnerability in real-world environments may be challenging; however, recent advancements in deep-learning and clustering technologies suggest that this attack could become a practical vulnerability. Therefore, we applied a countermeasure to $\mathsf{dGaussian}_\sigma$ to prevent these attacks. In the public key generation process of SMAUG-T, the $\mathsf{dGaussian}_\sigma$ function produces

integer intermediate values within the range of [-3, 3] when generating Gaussian errors. The significant hamming weight difference between positive and negative values distinguishes these values into two sets. (ex, {-3, -2, -1} / {0, 1, 2, 3}) With this distinction and linear algebraic approach, there is the possibility of recovering secret keys or reducing candidates.

Therefore, countermeasures are necessary. First, we consider masking techniques. However, designing a general random masking scheme efficiently in situations with numerous nonlinear bit-operations can be challenging and may incur significant overhead. For example, Krausz et al. [52] have recently proposed masking methods for the fixed hamming weight sampler; their efficiency is lacking, so we see it as future work. Hiding can be considered as another countermeasure. This attack involves logic that categorizes coefficients during the key generation process, making it difficult to distinguish which coefficients belong to which set is sufficient to respond effectively. Therefore, applying hiding would be more effective than masking. The errors are calculated in Figure 5 and then stored sequentially in each index. We applied the Fisher-Yates shuffle algorithm only to the corresponding loops and the loops where those values are utilized. In environments such as TLS, key generation is typically performed on servers with high-performance capabilities and operates less frequently than encryption. Therefore, such countermeasures will have a minimal impact on the cryptographic system.

D2 encoding and error reconciliation. During the KpqC round 1, Hee-Seok Kim reported the vulnerability related to power analysis caused by differences in the Hamming weight of the *mask* variable in the D2 encoding process. This attack was complemented in TiGER v2.1 by changing the *mask* variable to 1 and 0 and applying a countermeasure to minimize the Hamming weight difference. TiMER also prevents such vulnerability with the same countermeasure.

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