

# SMAUG: the Key Exchange Algorithm based on Module-LWE and Module-LWR

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**Abstract.** In this documentation, we introduce a new lattice-based post-quantum key encapsulation mechanism (KEM), a type of key exchange algorithm that we submit to the Korean Post-Quantum Cryptography Competition. Based on the hardness of the MLWE and MLWR problems, defined in module lattices, we design an efficient public key encryption (PKE) scheme **SMAUG.PKE** and KEM scheme **SMAUG.KEM**. With the choice of sparse secret, **SMAUG** achieves ciphertext sizes up to 12% and 9% smaller than Kyber (NIST’s 2022 selection) and Saber (NIST’s finalist), with much faster running time, up to 120% and 70%, respectively.

**Keywords:** Lattice-based Cryptography · Post-Quantum Cryptography · Key Encapsulation Mechanism · Module Learning With Errors · Module Learning With Roundings.

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This work is submitted to ‘Korean Post-Quantum Cryptography Competition’ ([www.kpqc.or.kr](http://www.kpqc.or.kr)).

## Changelog

**October 30, 2023 (version 2.0)** First, we update the hamming weight sampler HWT, which was not running in a constant-time due to the dependency on the number of hash calls. The new hamming weight sampler is a hybrid of the previous HWT algorithm, which was adopted from `SampleInBall` algorithm in `Dilithium`, and the constant weight word sampler [38]. We verified that the new sampler runs in a constant time, with a fixed number of hash calls. With the new hamming weight sampler and the partly optimized reference code, `SMAUG` is now 17% faster than previous version.

Second, we give an additionally security analysis for the choice of the approximate discrete Gaussian sampler. Using the Rényi divergence, it is theoretically guaranteed that the security loss comes from the approximation is minute.

Lastly, we give an additionally justification for the decryption failure probability against the state-of-the-art decryption failure attacks, asserting that the current failure probability of `SMAUG` is already low enough due to the attack scenarios.

**May 23, 2023 (version 1.0)** First, we update the python script for DFP computation as it was computing the decryption failure probability (DFP) wrongly. Note that the script was missing in the submission file, but included in our website. The parameter sets for NIST's security levels 3 and 5 were having higher DFPs then they were reported in the 1st round submission. As a result, the parameter sets are updated.

Second, we additionally compress the ciphertexts. As compression makes the error larger, we exploit the balance between the sizes and DFP.

Third, we put additional cost estimations on some algebraic and topological attacks: Arora-Ge [8], Coded-BKW [39], and Meet-LWE [51] attacks. We note that the previous parameter sets were all in a secure region against these attacks; however, for the new parameter sets, we aim to have more security margins. We put our code for estimating the cost of Meet-LWE attack in the python script.

Based on the above three updates, we changed our recommended parameter sets. As  $q = 1024$  is not available anymore for sufficient DFPs in the security levels 3 and 5, we move to  $q = 2048$  for those levels, resulting in slightly larger public key and secret key sizes. The ciphertext sizes are decreased by at most 96 bytes.

We also update the reference implementation to have a constant running time with much faster speed. It is uploaded to our website: `kpqc.cryptolab.co.kr/smaug`.

# 1 Introduction

SMAUG is an efficient post-quantum key encapsulation mechanism whose security is based on the hardness of the lattice problems. The IND-CPA security of SMAUG.PKE relies on the hardness of MLWE problem and MLWR problem, which implies the IND-CCA2 security of SMAUG.KEM.

Our SMAUG.KEM scheme follows the approaches in recent constructions of post-quantum KEMs such as Lizard [26] and RLizard [49]. SMAUG.KEM bases its security on the module variant lattice problems: the public key does not leak the secret key information by the hardness of MLWE problem, and the ciphertext protects sharing keys based on the hardness of MLWR problem. SMAUG consists of an underlying public key encryption (PKE) scheme SMAUG.PKE, which turns into SMAUG.KEM via Fujisaki-Okamoto transform.

## 1.1 Design rationale

The design rationale of SMAUG aims is to achieve small ciphertext and public key with low computational cost while maintaining security against various attacks. In more detail, we target the following practicality and security requirements considering real applications:

*Practicality:*

- Both the public key and ciphertext, especially the latter, which is transmitted more frequently, need to be short in order to minimize communication costs.
- As the key exchange protocol is frequently required on various personal devices, a KEM algorithm with low computational costs is more feasible than a high-cost one.
- A small secret key is desirable in restricted environments such as embedded or IoT devices since managing the secure zone is crucial to prevent physical attacks on secret key storage.

*Security:*

- The shared key should have a large enough entropy, at least  $\geq 256$  bits, to prevent Grover's search [37].
- Security should be concretely guaranteed concerning the attacks on the underlying assumptions, say lattice attacks.
- The low enough decryption failure probability (DFP) is essential to avoid the attacks boosting the failure and exploiting the decryption failures [28, 45].
- As KEMs are widely used in various devices and systems, countermeasures against implementation-specific attacks should also be considered. Especially combined with DFP, using error correction code (ECC) on the message to reduce decryption failures should be avoided since masking ECC against side-channel attacks is a very challenging problem.

**MLWE and MLWR.** SMAUG is constructed on the hardness of MLWE (Module-Learning with Errors) and MLWR (Module-Learning with Rounding) problems and follow the key structure of Lizard [26] and Ring-Lizard (RLizard) [49]. Since LWE problem has been a well-studied problem for the last two decades, there are many LWE-based schemes (e.g., Frodo [17]). Ring and module LWE problems are variants defined over structured lattices and regarded as hard as *LWE*. Many schemes base their security on RLWE/MLWE (e.g., NewHope [5], Kyber [16] and Saber [32]) for efficiency reasons. We also chose the module structure, which enables us to fine-tune security and efficiency in a much more scalable way, unlike standard and ring versions. Since MLWR problem is regarded as hard as MLWE problem unless we overuse the same secret to generate the samples [15], we chose to use MLWR samples for the encryption. By basing the security of encryption to MLWR, we reduce the ciphertext size by  $\log q / \log p$  than MLWE instances so that more efficient encryption and decryption are possible.

**Quantum Fujisaki-Okamoto transform.** SMAUG consists of key encapsulation mechanism SMAUG.KEM and public key encryption scheme SMAUG.PKE. On top of the PKE scheme, we construct the KEM scheme using the Fujisaki-Okamoto (FO) transform [35,36]. Line of works on FO transforms in the quantum random oracle model [14,42,46,52] make it possible to analyze the quantum security, i.e., in the quantum random oracle model (QROM). In particular, we use the FO transform with implicit rejection and no ciphertext contributions ( $\text{FO}_m^\chi$ ) following [44].

**Sparse secret key and ephemeral key.** We design the key generation algorithm based on MLWE problem using sparse secret. Based on the hardness reduction on the LWE problem using sparse secret [25], we use sparse ternary polynomials for the secret key and the ephemeral polynomial vectors. We take advantage of the sparsity, e.g., significantly smaller secret keys and faster multiplications. In particular, the small secret makes SMAUG more feasible in IoT devices having restricted resources.

**Choice of moduli.** All our moduli are powers of 2. This choice makes SMAUG enjoy faster encapsulation using simple bit shiftings, easy uniform sampling, and scalings. The power of 2 moduli makes it hard to apply Number Theoretic Transform (NTT) on the polynomial multiplications. However, small enough moduli and polynomial degrees enable SMAUG to achieve faster speed.

**Negligible decapsulation failures.** Since we base the security on the lattice problems, noise is inherent. The decryption result of a SMAUG.PKE ciphertext could be different from the original message but with negligible probability, say decryption failure probability (DFP). As other LWE/LWR-based KEMs, SMAUG has the message-independent DFP. We balance the sizes, DFP, and

security of SMAUG by fine-tuning the parameters. Trade-offs between them could give additional parameter sets for specific purposes.

We give estimated security and sizes for our parameter sets in Table 1. The complete parameter sets are given in Section 5. The security is estimated via lattice estimator [2] and our code. It is shown in core-SVP hardness. The DFP is calculated via a python script modified from Lizard and RLizard [43] and is reported in logarithm base two. The sizes are given in bytes. We include the security estimator of SMAUG in the reference code package on the team SMAUG website: [kqc.cryptolab.co.kr/smaug](http://kqc.cryptolab.co.kr/smaug).

| Parameters sets<br>Target security | SMAUG 128       | SMAUG 192        | SMAUG 256       |
|------------------------------------|-----------------|------------------|-----------------|
|                                    | 1               | 3                | 5               |
| $n$                                | 256             | 256              | 256             |
| $k$                                | 2               | 3                | 5               |
| $(q, p, p')$                       | (1024, 256, 32) | (2048, 256, 256) | (2048, 256, 64) |
| Classical core-SVP hardness        | 120.0           | 181.7            | 264.5           |
| Quantum core-SVP hardness          | 105.6           | 160.9            | 245.2           |
| Decryption failure probability     | -120            | -136             | -167            |
| Secret key size                    | 176             | 236              | 218             |
| Public key size                    | 672             | 1088             | 1792            |
| Ciphertext size                    | 672             | 1024             | 1472            |

Table 1: Security and sizes for our parameter sets.

## 1.2 Advantages and limitations

**Advantages** The security of SMAUG relies on the hardness of the lattice problems MLWE and MLWR, which enable balancing between security and efficiency. In terms of sizes, SMAUG has smaller ciphertext sizes compared to Kyber or Saber, which is the smallest ciphertext size among the recent practical lattice-based KEMs which avoid using the error correction codes. In terms of DFP, SMAUG achieves low enough DFP, which are similar to that of Saber. We do not use error correction code (ECC) to avoid side-channel attacks targeting decryption failures, to achieve masking-friendly implementation. Implementation-wise, encapsulation and decapsulation of SMAUG can be done efficiently. This makes SMAUG much easier to implement and secure against physical attacks. We give the constant-time C reference code, which proves the completeness and shows the efficiency of SMAUG.

### Limitations

- We use MLWR problem, which has been studied shorter than MLWE or LWE problems; however, it has a security reduction to MLWE. MLWE problem with a sparse secret has a similar issue but has been studied much longer and is used in various applications, e.g., homomorphic encryptions.

- As we use MLWE problem for the secret key security, larger public key sizes than Saber are inherent. It can be seen as a trade-off between the public key size versus performance with a smaller secret key size.

## 2 Preliminaries

### 2.1 Notation

We denote matrices with bold and upper case letters (e.g.,  $\mathbf{A}$ ) and vectors with bold type and lower case letters (e.g.,  $\mathbf{b}$ ). Unless otherwise stated, the vector is a column vector.

We define a polynomial ring  $\mathcal{R} = \mathbb{Z}[x]/(x^n + 1)$  where  $n$  is a power of 2 integers and denote a quotient ring by  $\mathcal{R}_q = \mathbb{Z}[x]/(q, x^n + 1) = \mathbb{Z}_q[x]/(x^n + 1)$  for a positive integer  $q$ . For an integer  $\eta$ , we denote the set of polynomials of degree less than  $n$  with coefficients in  $[-\eta, \eta] \cap \mathbb{Z}$  as  $S_\eta$ . Let  $\tilde{S}_\eta$  be a set of polynomials of degree less than  $n$  with coefficients in  $[-\eta, \eta] \cap \mathbb{Z}$ . We denote a discrete Gaussian distribution with standard deviation  $\sigma$  as  $\mathcal{D}_{\mathbb{Z}, \sigma}$ . We define Rényi divergence of order  $\alpha$  between to probability distributions  $P$  and  $Q$  such that  $\text{Supp}(P) \subseteq \text{Supp}(Q)$  as  $R_\alpha(P\|Q) = \left( \sum_{x \in \text{Supp}(P)} \frac{P(x)^\alpha}{Q(x)^{\alpha-1}} \right)^{1/(\alpha-1)}$ , where  $\text{Supp}(D)$  for a distribution  $D$  is defined as  $\text{Supp}(D) = \{x \in D : D(x) \neq 0\}$ .

### 2.2 Lattice assumptions

We first define some well-known lattice assumptions *MLWE* and *MLWR* on the structured Euclidean lattices.

**Definition 1 (Decision-*MLWE* $_{n,q,k,\ell,\eta}$ ).** For positive integers  $q, k, \ell, \eta$  and the dimension  $n$  of  $\mathcal{R}$ , we say that the advantage of an adversary  $\mathcal{A}$  solving the decision-*MLWE* $_{n,q,k,\ell,\eta}$  problem is

$$\text{Adv}_{n,q,k,\ell,\eta}^{\text{MLWE}}(\mathcal{A}) = \left| \Pr [b = 1 \mid \mathbf{A} \leftarrow \mathcal{R}_q^{k \times \ell}; \mathbf{b} \leftarrow \mathcal{R}_q^k; b \leftarrow \mathcal{A}(\mathbf{A}, \mathbf{b})] \right. \\ \left. - \Pr [b = 1 \mid \mathbf{A} \leftarrow \mathcal{R}_q^{k \times \ell}; (\mathbf{s}, \mathbf{e}) \leftarrow S_\eta^\ell \times S_\eta^k; b \leftarrow \mathcal{A}(\mathbf{A}, \mathbf{A} \cdot \mathbf{s} + \mathbf{e})] \right|$$

**Definition 2 (Decision-*MLWR* $_{n,p,q,k,\ell,\eta}$ ).** For positive integers  $p, q, k, \ell, \eta$  with  $q \geq p \geq 2$  and the dimension  $n$  of  $\mathcal{R}$ , we say that the advantage of an adversary  $\mathcal{A}$  solving the decision-*MLWR* $_{n,p,q,k,\ell,\eta}$  problem is

$$\text{Adv}_{n,p,q,k,\ell,\eta}^{\text{MLWE}}(\mathcal{A}) = \left| \Pr [b = 1 \mid \mathbf{A} \leftarrow \mathcal{R}_p^{k \times \ell}; \mathbf{b} \leftarrow \mathcal{R}_q^k; b \leftarrow \mathcal{A}(\mathbf{A}, \mathbf{b})] \right. \\ \left. - \Pr [b = 1 \mid \mathbf{A} \leftarrow \mathcal{R}_q^{k \times \ell}; \mathbf{s} \leftarrow S_\eta^\ell; b \leftarrow \mathcal{A}(\mathbf{A}, \lfloor p/q \cdot \mathbf{A} \cdot \mathbf{s} \rfloor)] \right|$$

### 2.3 Public key encryption and key encapsulation mechanism

We then recap the formalisms of PKE and KEM.

**Definition 3 (PKE).** A public key encryption scheme is a tuple of PPT algorithms  $\text{PKE} = (\text{KeyGen}, \text{Enc}, \text{Dec})$  with the following specifications:

- **KeyGen:** a probabilistic algorithm that outputs a public key  $\text{pk}$  and a secret key  $\text{sk}$ ;

- **Enc**: a probabilistic algorithm that takes as input a public key  $\text{pk}$  and a message  $\mu$  and outputs a ciphertext  $\text{ct}$ ;
- **Dec**: a deterministic algorithm that takes as input a secret key  $\text{sk}$  and a ciphertext  $\text{ct}$  and outputs a message  $\mu$ .

Let  $0 < \delta < 1$ . We say that it is  $(1 - \delta)$ -correct if for any  $(\text{pk}, \text{sk})$  generated from  $\text{KeyGen}$  and  $\mu$ ,

$$\Pr[\text{Dec}(\text{sk}, \text{Enc}(\text{pk}, \mu)) \neq \mu] \leq \delta,$$

where the probability is taken over the randomness of the encryption algorithm. We call the above probability decryption failure probability (DFP). In addition, we say that it is correct in the (Q)ROM if the probability is taken over the randomness of the (quantum) random oracle, modeling the hash function.

**Definition 4 (KEM).** A key encapsulation mechanism scheme is a tuple of PPT algorithms  $\text{KEM} = (\text{KeyGen}, \text{Encap}, \text{Decap})$  with the following specifications:

- **KeyGen**: a probabilistic algorithm that outputs a public key  $\text{pk}$  and a secret key  $\text{sk}$ ;
- **Encap**: a probabilistic algorithm that takes as input a public key  $\text{pk}$  and outputs a sharing key  $K$  and a ciphertext  $\text{ct}$ ;
- **Decap**: a deterministic algorithm that takes input a secret key  $\text{sk}$  and a ciphertext  $\text{ct}$  and outputs a sharing key  $K$ .

The correctness of KEM is defined similarly to that of PKE.

We give the advantage function with respect to the attacks against PKE, namely the INDistinguishability under Chosen Plaintext Attacks (IND-CPA).

**Definition 5 (IND-CPA security of PKE).** For a (quantum) adversary  $\mathcal{A}$  against a public key encryption scheme  $\text{PKE} = (\text{KeyGen}, \text{Enc}, \text{Dec})$ , we define the IND-CPA advantage of  $\mathcal{A}$   $(\mathcal{A}_1, \mathcal{A}_2)$  as follows:

$$\text{Adv}_{\text{PKE}}^{\text{IND-CPA}}(\mathcal{A}) = \left| \Pr_{(\text{pk}, \text{sk})} \left[ b = b' \mid \begin{array}{l} (\mu_0, \mu_1, st) \leftarrow \mathcal{A}_1(\text{pk}); \ b \leftarrow \{0, 1\}; \\ \text{ct} \leftarrow \text{Enc}(\text{pk}, \mu_b); \ b' \leftarrow \mathcal{A}_2(\text{pk}, \text{ct}, st) \end{array} \right] - \frac{1}{2} \right|.$$

The probability is taken over the randomness of  $\mathcal{A}$  and  $(\text{pk}, \text{sk}) \leftarrow \text{KeyGen}(1^\lambda)$ .

We then define two advantage functions with respect to the attacks against KEM, namely the INDistinguishability under Chosen Plaintext Attacks (IND-CPA) as in PKE and the INDistinguishability under (adaptively) Chosen Ciphertext Attacks (IND-CCA).

**Definition 6 (IND-CPA and IND-CCA security of KEM).** For a (quantum) adversary  $\mathcal{A}$  against a key encapsulation mechanism  $\text{KEM} = (\text{KeyGen}, \text{Encap}, \text{Decap})$ , we define the IND-CPA advantage of  $\mathcal{A}$  as follows:

$$\text{Adv}_{\text{KEM}}^{\text{IND-CPA}}(\mathcal{A}) = \left| \Pr_{(\text{pk}, \text{sk})} \left[ b = b' \mid \begin{array}{l} b \leftarrow \{0, 1\}; \ (K_0, \text{ct}) \leftarrow \text{Encap}(\text{pk}); \\ K_1 \leftarrow \mathcal{K}; \ b' \leftarrow \mathcal{A}(\text{pk}, \text{ct}, K_b) \end{array} \right] - \frac{1}{2} \right|.$$

The probability is taken over the randomness of  $\mathcal{A}$  and  $(\text{pk}, \text{sk}) \leftarrow \text{KeyGen}(1^\lambda)$ . The IND-CCA advantage of  $\mathcal{A}$  is defined similarly except that the adversary can query  $\text{Decap}(\text{sk}, \cdot)$  oracle on any ciphertext  $\text{ct}' (\neq \text{ct})$ .



We can then define the (quantum) security notions of PKE and KEM in the (Q)ROM.

**Definition 7 ((Q)ROM security of PKE and KEM).** *For  $T, \epsilon > 0$ , we say that a scheme  $\mathcal{S} \in \{\text{PKE}, \text{KEM}\}$  is  $(T, \epsilon)$ -ATK secure in the (Q)ROM if for any (quantum) adversary  $\mathcal{A}$  with runtime  $\leq T$  given classical access to  $\mathcal{O}$  and (quantum) access to a random oracle  $H$ , it holds that  $\text{Adv}_{\mathcal{S}}^{\text{ATK}}(\mathcal{A}) < \epsilon$ , where*

$$\mathcal{O} = \begin{cases} \text{Enc} & \text{if } \mathcal{S} = \text{PKE and } \text{ATK} \in \{\text{OW-CPA}, \text{IND-CPA}\}, \\ \text{Encap} & \text{if } \mathcal{S} = \text{KEM and } \text{ATK} = \text{IND-CPA}, \\ \text{Encap, Decap}(\text{sk}, \cdot) & \text{if } \mathcal{S} = \text{KEM and } \text{ATK} = \text{IND-CCA}. \end{cases}$$

## 2.4 Fujisaki-Okamoto transform

Fujisaki and Okamoto proposed a novel generic transform [35, 36] that turns a weakly secure PKE scheme into a strongly secure PKE scheme in the Random Oracle Model (ROM), and various variants have been proposed to deal with tightness, non-correct PKEs, and in the quantum setting, i.e., QROM. Here, we recall the FO transformation for KEM as introduced by Dent [30] and revisited by Hofheinz et al. [42], Bindel et al. [13], and Hövelmanns et al. [44].

The original FO transforms  $\text{FO}_m^\perp$  constructs a KEM from a deterministic PKE, i.e., a de-randomized version. The encapsulation randomly samples a message  $m$  and uses the message’s hash value  $G(m)$  as randomness for encryption, generating a ciphertext. The sharing key  $K = H(m)$  is generated by hashing (with different hash functions) the message. In the decapsulation, it first decrypts the ciphertext and recovers the message,  $m'$ . If it fails to decrypt, it outputs  $\perp$ . If the “re-encryption” of the recovered message is not equal to the received ciphertext, it also outputs  $\perp$ . The sharing key can be generated by hashing the recovered message.

In the quantum setting, however, the FO transform with “implicit rejection” ( $\text{FO}_m^\perp$ ) has a tighter security proof than the original version, which implicitly outputs a pseudo-random sharing key if the re-encryption fails. We recap the QROM proof of Bindel et al. [13] allowing the KEMs constructed over non-perfect PKEs to have IND-CCA security.

**Theorem 1 ([13], Theorem 1 & 2).** *Let  $G$  and  $H$  be quantum-accessible random oracles, and the deterministic PKE is  $\epsilon$ -injective. Then the advantage of IND-CCA attacker  $\mathcal{A}$  with at most  $Q_{\text{Dec}}$  decryption queries and  $Q_G$  and  $Q_H$  hash queries at depth at most  $d_G$  and  $d_H$ , respectively, is*

$$\begin{aligned} \text{Adv}_{\text{KEM}}^{\text{IND-CCA}}(\mathcal{A}) \leq & 2\sqrt{(d_G + 2) \left( \text{Adv}_{\text{PKE}}^{\text{IND-CPA}}(\mathcal{B}_1) + 8(Q_G + 1)/|\mathcal{M}| \right)} \\ & + \text{Adv}_{\text{PKE}}^{\text{DF}}(\mathcal{B}_2) + 4\sqrt{d_H Q_H / |\mathcal{M}|} + \epsilon, \end{aligned}$$

where  $\mathcal{B}_1$  is an IND-CPA adversary on PKE and  $\mathcal{B}_2$  is an adversary against finding a decryption failing ciphertext, returning at most  $Q_{\text{Dec}}$  ciphertexts.

### 3 Design choices

In this section, we explain the design choices for SMAUG.

#### 3.1 MLWE public key and MLWR ciphertext

One of the core designs of SMAUG uses the MLWE hardness for its secret key security and MLWR hardness for its message security. This choice is adapted from Lizard and RLizard, which use LWE/LWR and RLWE/RLWR, respectively. Using both LWE and LWR variant problems makes the conceptual security distinction between the secret key and the ephemeral sharing key: a more conservative secret key with more efficient en/decapsulations. This can be viewed as a trade-off between “conservative” and “efficient” designs. Combined with the sparse secret, bringing the LWE-based key generation to the LWR-based scheme enables balancing the speed and the DFP.

**3.1.1 Public key.** Public key of SMAUG consists of a vector  $\mathbf{b}$  over a polynomial ring  $\mathcal{R}_q$  and a matrix  $\mathbf{A}$ , which can be viewed as an MLWE sample,

$$(\mathbf{A}, \mathbf{b} = -\mathbf{A}^\top \mathbf{s} + \mathbf{e}) \in \mathcal{R}_q^{k \times k} \times \mathcal{R}_q^k,$$

where  $\mathbf{s}$  is a ternary secret polynomial with hamming weight  $h_s$ , and  $\mathbf{e}$  is an error sampled from discrete Gaussian distribution with standard deviation  $\sigma$ . We hereby specify the uniform matrix sampling algorithm for  $\mathbf{A} \in \mathcal{R}_q^{k \times k}$  in Figure 1. It is adapted from the pseudorandom generator `gen` in Saber [29].

|   |   |
|---|---|
| <code>expandA(seed):</code><br>1: <code>buf ← XOF(seed)</code><br>2: <b>for</b> $i$ from 0 to $k - 1$ <b>do</b><br>3: <code>A[i] = bytes_to_Rq(buf + polybytes · i)</code><br>4: <b>return</b> <code>A</code> | $\triangleright \text{seed} \in \{0, 1\}^{256}$<br><br><br>$\triangleright \text{Convert to ring elements}$ |
|---|---|

Fig. 1: Uniform random matrix sampler, `expandA`.

We note that the public key of SMAUG consists of  $\mathbf{b}$  and the seed of  $\mathbf{A}$ .

**3.1.2 Ciphertext.** The ciphertext of SMAUG is a tuple of a vector  $\mathbf{c}_1 \in \mathcal{R}_p^k$  and a polynomial  $c_2 \in \mathcal{R}_p$ . The ciphertext is generated by multiplying a random vector  $\mathbf{r}$  to the public key; then it is scaled and rounded as,

$$\mathbf{c} = \begin{bmatrix} \mathbf{c}_1 \\ c_2 \end{bmatrix} = \left\lfloor \frac{p}{q} \cdot \begin{pmatrix} \mathbf{A} \\ \mathbf{b}^\top \end{pmatrix} \cdot \mathbf{r} \right\rfloor + \frac{p}{t} \cdot \begin{bmatrix} 0 \\ \mu \end{bmatrix},$$

Along with the public key, it can be treated as an MLWR sample added by a scaled message as  $(\mathbf{A}', \lfloor p/q \cdot \mathbf{A}' \cdot \mathbf{r} \rfloor) + (0, \mu')$ , where  $\mathbf{A}'$  is a concatenated matrix of  $\mathbf{A}$  and  $\mathbf{b}^\top$ .

The ciphertext can be further compressed by scaling the second component  $c_2$  by  $p'/p$ , resulting in a shorter ciphertext but a larger error. We note that the public key can be compressed with the same technique. However, it introduces a more significant error, so we do not compress the public key in SMAUG.

### 3.2 Sparse secret

For the randomnesses  $\mathbf{s}$  and  $\mathbf{r}$ , we use the sparse ternary distribution. In the following, we will discuss the advantages of the sparse secret and give the sampling algorithm.

**3.2.1 Advantage of using sparse secret** The sparse secret is widely used in homomorphic encryption to reduce the noise propagation during the homomorphic operations [19, 24, 40] and to speed up the computations. As the lattice-based KEM schemes have inherent decryption error from LWE or LWR noise, the sparse secret can lower this decryption error and also improve the performance of KEMs.

Concretely, the decryption error can be expressed as  $\langle \mathbf{e}, \mathbf{r} \rangle + \langle \mathbf{e}_1, \mathbf{s} \rangle + e_2$ , where  $\mathbf{s}$  is a secret key,  $\mathbf{r}$  is a randomness used for encryption,  $\mathbf{e} \leftarrow \chi_{pk}^k$  is a noise added in public key, and  $(\mathbf{e}_1, e_2) \leftarrow \chi_{ct}^{k+1}$  is a noise added in ciphertext. As the vectors  $\mathbf{r}$  and  $\mathbf{s}$  are binary (ternary, resp.), each coefficient of the decryption error is an addition (signed addition, resp.) of  $h_r$  variables from  $\chi_{pk}$  and  $h_s + 1$  variables from  $\chi_{ct}$ . The magnitude of the decryption error depends greatly on the Hamming weights  $h_r$  and  $h_s$ , and thus, we can take advantage of the sparse secrets.

Other major advantages of sparse secrets include reducing the secret key size and enabling fast polynomial multiplication. As the coefficients of the secret key are sparse with a fixed hamming weight, we can store only the information of the nonzero coefficients. We can further use this structure for the polynomial multiplications, which we will describe in Section 3.4.

On the other hand, as the sparse secret reduces the secret key entropy, the hardness of the lattice problem may be decreased. For the security of LWE problem using sparse secret, a series of works have been done, including [25] for asymptotic security based on the reductions to worst-case lattice problems, and [12, 33, 54] for concrete security. Independent of the secret distribution, the module variant (MLWE) is regarded as hard as LWE problem with appropriate parameters, including a smaller modulus. We also exploit the reductions from ordinary MLWE to MLWE using sparse secret or small errors [20]. The MLWR problem also has a simple reduction from MLWE independent of the secret distribution, and its concrete security is heuristically discussed in [29].

Since SMAUG uses a sparse secret key  $\mathbf{s}$  and a sparse randomness  $\mathbf{r}$ , the security of SMAUG is based on the hardness of MLWE and MLWR problems using

sparse secret. For the specific parameters, we exploit the lattice-estimator [2], which covers most of the recent lattice attacks, and also consider some attacks not included in the estimator. Using a smaller modulus, SMAUG can maintain high security, as in Kyber or Saber.

**3.2.2 Hamming weight sampler** Our hamming weight sampler,  $\text{HWT}_h$ , is a hybrid of the `SampleInBall` algorithm in Dilithium [31] and the CWW (constant weight word) sampler in BIKE [38], which have a constant running time. A detailed algorithm is given in Figure 2, which samples a ternary polynomial vector having a hamming weight of  $h$ .

|   |   |
|---|---|
| $\text{HWT}_h(\text{seed})$ :   | $\triangleright \text{seed} \in \{0, 1\}^{256}$           |
| 1: $\text{idx} = 0$   |   |
| 2: $\text{buf} \leftarrow \text{XOF}(\text{seed})$  |   |
| 3: <b>for</b> $i$ from $n - h$ to $n - 1$ <b>do</b>   |   |
| 4: $\text{degree} = (\text{buf}[\text{idx}] \cdot (i + 1)) \gg 32$                                      | $\triangleright \text{buf}[\text{idx}] \in \{0, 1\}^{32}$ |
| 5: $\text{res}[i] = \text{res}[\text{degree}]$  |   |
| 6: $\text{res}[\text{degree}] = \text{buf}[h + (\text{idx} \gg 4)] \gg (\text{idx} \wedge 0\text{x0f})$ |   |
| 7: $\text{res}[\text{degree}] = (\text{buf}[\text{degree}] \wedge 0\text{x02}) - 1$                     |   |
| 8: $\text{idx} = \text{idx} + 1$  |   |
| 9: <b>return</b> $\text{convToIdx}(\text{res})$   | $\triangleright$ Storing the indexes                      |

Fig. 2: Hamming weight sampler,  $\text{HWT}_h$ .

### 3.3 Discrete Gaussian noise

**3.3.1 Using approximate discrete Gaussian noise** Our design choice for the noise distribution in MLWE follows the conventional discrete Gaussian distribution, but with approximated CDTs following the approaches in FrodoKEM [17]. As a result, we use a discrete Gaussian noise for the public key generation, which is approximated to a narrow distribution. As this approximated discrete Gaussian noise is used only for the public key, we can efficiently bound the security loss. We give a theoretical justification based on Rényi divergence to guarantee the security of SMAUG, considering the narrow discrete Gaussian noise.

In SMAUG, the narrow discrete Gaussian noise is used only for the public key generation. So the difference in the noise distribution only affects the distinguishing advantage between the games  $G_2$  and  $G_3$  in the proof of Theorem 4. Then the bound for the distinguishing advantage can also be expressed as

$$\left( \text{Adv}_{n,q,k,k,\mathcal{D}_{\mathbb{Z},\sigma}}^{\text{MLWE}}(\mathcal{B}_2) \cdot R_\alpha(\text{dGaussian}_\sigma \| \mathcal{D}_{\mathbb{Z},\sigma})^{nk} \right)^{1-1/\alpha},$$

assuming the pseudorandomness of  $\mathbf{dGaussian}_\sigma$ . This is due to Lemma 5.5 in [4]. We note that the key generation calls  $\mathbf{dGaussian}$  only  $nk$  times and that the public key is generated only once.

The bound for our typical parameter set (see Section 5.2) for security level 1 (levels 3 and 5, resp.) increases from  $2^{-120.0}$  ( $2^{-181.7}$  and  $2^{-264.5}$ , resp.) to  $2^{-118.2}$  ( $2^{-176.9}$  and  $2^{-260.2}$ , resp.) with  $\alpha = 200$  (75 and 200, resp.). Opposed to the estimated security based on the bound  $\text{Adv}_{n,q,k,k,\mathbf{dGaussian}_\sigma}^{\text{MLWE}}(\mathcal{B}_2)$  given in Section 5.2, this new bound provides a more conservative security preventing some possible future attacks that target the noise distribution.

By using one more bit for  $\mathbf{dGaussian}$  algorithm, we can decrease the advantage to  $2^{-119.6}$  ( $2^{-181.2}$  and  $2^{-263.6}$ , resp.) with  $\alpha = 500$ . This modification will slightly decrease only the speed of key generation by less than 1.1x.

We note that the narrow Gaussian noise is already taken into account when estimating the concrete security (given in Section 5.2) using the explained estimators. The analysis given here provides a more conservative security, preventing some possible future attacks that target the noise distribution. We also note that in the core-SVP methodology, we only focus on the estimated attack cost of the underlying MLWE and MLWR problems, not based on the security reductions (as done in most of the NIST-submitted schemes) for a fair comparison to Kyber or Saber.

**3.3.2  $\mathbf{dGaussian}$  sampler** We construct  $\mathbf{dGaussian}$ , a constant-time approximate discrete Gaussian noise sampler, upon a Cumulative Distribution Table (CDT) but is not used during sampling, as it is expressed with bit operations. We first scale the discrete Gaussian distribution and make a CDT approximating the discrete Gaussian distribution. We choose an appropriate scaling factor based on the analysis in [17, 48] using Rényi divergence. We then deploy the Quine-McCluskey method<sup>5</sup> and apply logic minimization technique on the CDT. As a result, even though our  $\mathbf{dGaussian}$  is constructed upon CDT, it is expressed with bit operations and is constant-time. The algorithms are easily parallelizable and suitable for IoT devices as their memory requirement is low.

We describe  $\mathbf{dGaussian}$  with  $\sigma = 1.0625$  in Figure 3 and  $\sigma = 1.453713$  in Figure 4.

### 3.4 Polynomial multiplication using sparsity

SMAUG uses the power-of-two moduli to ease the correct scaling and roundings. However, this makes the polynomial multiplications hard to benefit from Number Theoretic Transform (NTT). To address this issue, we propose a new polynomial multiplication that takes advantage of sparsity, which we adapt from [1, 49]. Our new multiplication, given in Figure 5, is constant-time and is faster than the previous approach. We also use a similar secret storing technique as RLizard, where only the degrees of non-zero coefficients are stored in the secret key and directly used in polynomial multiplications.

<sup>5</sup> We use the python package, from <https://github.com/dreylago/logicmin>.

**dGaussian<sub>1.0625</sub>**( $x$ ):

**Require:**  $x = x_0x_1x_2x_3x_4x_5x_6x_7x_8x_9 \in \{0, 1\}^{10}$

1:  $s = s_1s_0 = 00 \in \{0, 1\}^2$

2:  $s_0 = x_0x_1x_2x_3x_4x_5x_7\overline{x_8}$

3:  $s_0 += (x_0x_3x_4x_5x_6x_8) + (x_1x_3x_4x_5x_6x_8) + (x_2x_3x_4x_5x_6x_8)$

4:  $s_0 += (\overline{x_2x_3x_6x_8}) + (\overline{x_1x_3x_6x_8})$

5:  $s_0 += (x_6x_7\overline{x_8}) + (\overline{x_5x_6x_8}) + (\overline{x_4x_6x_8}) + (\overline{x_7x_8})$

6:  $s_1 = (x_1x_2x_4x_5x_7x_8) + (x_3x_4x_5x_7x_8) + (x_6x_7x_8)$

7:  $s = (-1)^{x_9} \cdot s$

$\triangleright \cdot$  is the arithmetic multiplication

8: **return**  $s$

Fig. 3: Discrete Gaussian sampler with  $\sigma = 1.0625$ , **dGaussian <sub>$\sigma$</sub>** .

**dGaussian<sub>1.453713</sub>**( $x$ ):

**Require:**  $x = x_0x_1x_2x_3x_4x_5x_6x_7x_8x_9x_{10} \in \{0, 1\}^{11}$

1:  $s = s_2s_1s_0 = 000 \in \{0, 1\}^3$

2:  $s_0 = (x_0x_1x_2x_3x_5x_7x_8) + (x_1x_2x_3x_5\overline{x_6x_7x_9}) + (\overline{x_1x_2x_3x_6x_7x_8})$

3:  $s_0 += (\overline{x_1x_2x_3x_5x_8x_9}) + (\overline{x_0x_2x_3x_5x_8x_9})$

4:  $s_0 += (x_4x_5\overline{x_6x_7x_9}) + (x_3x_4x_8\overline{x_9}) + (\overline{x_5x_6x_7x_8}) + (\overline{x_4x_6x_7x_8}) + (\overline{x_4x_5x_8x_9})$

5:  $s_0 += (x_5x_8\overline{x_9}) + (x_6x_8\overline{x_9}) + (x_7x_8\overline{x_9}) + (\overline{x_7x_8x_9}) + (\overline{x_6x_8x_9})$

6:  $s_1 = (x_0x_1x_4\overline{x_5x_6x_7x_9}) + (x_2x_4\overline{x_5x_6x_7x_9}) + (x_3x_4\overline{x_5x_6x_7x_9}) + (x_5x_6x_7\overline{x_8x_9})$

7:  $s_1 += (\overline{x_1x_2x_3x_8x_9}) + (\overline{x_7x_8x_9}) + (\overline{x_6x_8x_9}) + (\overline{x_5x_8x_9}) + (\overline{x_4x_8x_9})$

8:  $s_2 = (x_1x_4x_5x_6x_7x_8x_9) + (x_2x_4x_5x_6x_7x_8x_9) + (x_3x_4x_5x_6x_7x_8x_9)$

9:  $s = (-1)^{x_{10}} \cdot s$

$\triangleright \cdot$  is the arithmetic multiplication

10: **return**  $s$

Fig. 4: Discrete Gaussian sampler with  $\sigma = 1.453713$ , **dGaussian <sub>$\sigma$</sub>** .

### 3.5 FO transform, $\text{FO}_m^\chi$

We construct SMAUG upon the FO transform with implicit rejection and without ciphertext contribution to the sharing key generation, say  $\text{FO}_m^\chi$ . This choice makes the encapsulation and decapsulation algorithm efficient since the sharing key can be directly generated from a message. The public key is additionally fed into the hash function with the message to avoid multi-target decryption failure attacks. The IND-CCA security of the resulting KEM in the QROM is well-studied in [13, 42, 44].

|   |  |
|---|--|
| <pre> poly_mult_add(<math>a, b, \text{neg\_start}</math>): 1: <math>c = 0</math> 2: <b>for</b> <math>i</math> from 0 to <math>\text{neg\_start} - 1</math> <b>do</b> 3:   <math>\text{degree} = b[i]</math> 4:   <b>for</b> <math>j</math> from 0 to <math>n - 1</math> <b>do</b> 5:     <math>c[\text{degree} + j] = c[\text{degree} + j] + a[j];</math> 6: <b>for</b> <math>i</math> from <math>\text{neg\_start}</math> to <math>\text{len}(b) - 1</math> <b>do</b> 7:   <math>\text{degree} = b[i]</math> 8:   <b>for</b> <math>j</math> from 0 to <math>n - 1</math> <b>do</b> 9:     <math>c[\text{degree} + j] = c[\text{degree} + j] - a[j];</math> 10: <b>for</b> <math>j</math> from 0 to <math>n - 1</math> <b>do</b> 11:   <math>c[j] = c[j] - c[n + j];</math> 12: <b>return</b> <math>c</math> </pre> | $\triangleright a \in \mathcal{R}_q, b \in \mathcal{S}_\eta$ |
|---|--|

Fig. 5: Polynomial multiplication using sparsity.

## 4 The SMAUG

### 4.1 Specification of SMAUG.PKE

We now describe the public key encryption scheme SMAUG.PKE in Figure 6 with the following building blocks:

- Extendable output function XOF for generating  $\text{seed}_A$ ,  $\text{seed}_{sk}$ , and  $\text{seed}_e$ ,
- Uniform random matrix sampler  $\text{expandA}$  for deriving  $\mathbf{A}$  from  $\text{seed}_A$ ,
- Discrete Gaussian sampler  $\text{dGaussian}_\sigma$  for deriving a MLWE noise  $\mathbf{e}$  with standard deviation  $\sigma$  from  $\text{seed}_e$ ,
- Hamming weight sampler  $\text{HWT}_h$  for deriving a sparse ternary  $\mathbf{s}$  (resp.  $\mathbf{r}$ ) with hamming weight  $h = h_s$  (resp.  $h = h_r$ ) from  $\text{seed}_{sk}$  (resp.  $\text{seed}_r$ ).

|   |   |
|---|---|
| <b>KeyGen(<math>1^\lambda</math>):</b><br>1: $\text{seed} \leftarrow \{0, 1\}^{256}$<br>2: $(\text{seed}_A, \text{seed}_{sk}, \text{seed}_e) \leftarrow \text{XOF}(\text{seed})$<br>3: $\mathbf{A} \leftarrow \text{expandA}(\text{seed}_A) \in \mathcal{R}_q^{k \times k}$<br>4: $\mathbf{s} \leftarrow \text{HWT}_{h_s}(\text{seed}_{sk}) \in S_\eta^k$<br>5: $\mathbf{e} \leftarrow \text{dGaussian}_\sigma(\text{seed}_e) \in \mathcal{R}^k$<br>6: $\mathbf{b} = -\mathbf{A}^\top \cdot \mathbf{s} + \mathbf{e} \in \mathcal{R}_q^k$<br>7: <b>return</b> $\text{pk} = (\text{seed}_A, \mathbf{b})$ , $\text{sk} = \mathbf{s}$ |   |
| <b>Enc(<math>\text{pk}, \mu; \text{seed}_r</math>):</b><br>1: $\mathbf{A} = \text{expandA}(\text{seed}_A)$<br>2: <b>if</b> $\text{seed}_r$ is not given <b>then</b> $\text{seed}_r \leftarrow \{0, 1\}^{256}$<br>3: $\mathbf{r} \leftarrow \text{HWT}_{h_r}(\text{seed}_r) \in S_\eta^k$<br>4: $\mathbf{c}_1 = \lfloor p/q \cdot \mathbf{A} \cdot \mathbf{r} \rfloor \in \mathcal{R}_p^k$<br>5: $c_2 = \lfloor p'/q \cdot \langle \mathbf{b}, \mathbf{r} \rangle + p'/t \cdot \mu \rfloor \in \mathcal{R}_{p'}$<br>6: <b>return</b> $\text{ct} = (\mathbf{c}_1, c_2)$   | $\triangleright \text{pk} = (\text{seed}_A, \mathbf{b}), \mu \in \mathcal{R}_t$ |
| <b>Dec(<math>\text{sk}, \mathbf{c}</math>):</b><br>1: $\mu' = \lfloor t/p \cdot \langle \mathbf{c}_1, \mathbf{s} \rangle + t/p' \cdot c_2 \rfloor \in \mathcal{R}_t$<br>2: <b>return</b> $\mu'$   | $\triangleright \text{sk} = \mathbf{s}, \mathbf{c} = (\mathbf{c}_1, c_2)$       |

Fig. 6: Description of SMAUG.PKE

We then prove the completeness of SMAUG.PKE.

**Theorem 2 (Completeness of SMAUG.PKE).** *Let  $\mathbf{A}$ ,  $\mathbf{b}$ ,  $\mathbf{s}$ ,  $\mathbf{e}$ , and  $\mathbf{r}$  are defined as in Figure 6. Let the moduli  $t$ ,  $p$ ,  $p'$ , and  $q$  satisfy  $t \mid p \mid q$  and  $t \mid p' \mid q$ . Let  $\mathbf{e}_1 \in \mathcal{R}_\mathbb{Q}^k$  and  $e_2 \in \mathcal{R}_\mathbb{Q}$  be the rounding errors introduced from the scalings and roundings of  $\mathbf{A} \cdot \mathbf{r}$  and  $\mathbf{b}^T \cdot \mathbf{r}$ . That is,  $\mathbf{e}_1 = \frac{q}{p}(\lfloor \frac{p}{q} \cdot \mathbf{A} \cdot \mathbf{r} \rfloor \bmod p) - (\mathbf{A} \cdot \mathbf{r} \bmod q)$  and  $e_2 = \frac{q}{p'}(\lfloor \frac{p'}{q} \cdot \langle \mathbf{b}, \mathbf{r} \rangle \rfloor \bmod p') - (\langle \mathbf{b}, \mathbf{r} \rangle \bmod q)$ . Let  $\delta =$*



$\Pr [\|\langle \mathbf{e}, \mathbf{r} \rangle + \langle \mathbf{e}_1, \mathbf{s} \rangle + e_2\|_\infty > \frac{q}{2t}]$ , where the probability is taken over the randomness of the encryption. Then **SMAUG.PKE** in Figure 6 is  $(1 - \delta)$ -correct. That is, for every message  $\mu$  and every key-pair  $(\mathbf{pk}, \mathbf{sk})$  returned by  $\text{KeyGen}(1^\lambda)$ , the decryption fails with a probability less than  $\delta$ .

*Proof.* By the definition of  $\mathbf{e}_1$  and  $e_2$ , it holds that  $\mathbf{c}_1 = \frac{p}{q} \cdot (\mathbf{A} \cdot \mathbf{r} + \mathbf{e}_1) \pmod{p}$  and  $c_2 = \frac{p'}{q} \cdot (\langle \mathbf{b}, \mathbf{r} \rangle + e_2) + \frac{p'}{t} \cdot \mu \pmod{p'}$ , where the coefficients of  $\mathbf{e}_1$  and  $e_2$  are in  $\mathbb{Z} \cap (-\frac{q}{2p}, \frac{q}{2p}]$  and  $\mathbb{Z} \cap (-\frac{q}{2p'}, \frac{q}{2p'}]$ , respectively. Thus, the decryption of the ciphertext  $(\mathbf{c}_1, c_2)$  can be written as

$$\begin{aligned} \left\lfloor \frac{t}{p} \cdot \langle \mathbf{c}_1, \mathbf{s} \rangle + \frac{t}{p'} \cdot c_2 \right\rfloor \pmod{t} &= \left\lfloor \frac{t}{q} (\langle \mathbf{A} \cdot \mathbf{r}, \mathbf{s} \rangle + \langle \mathbf{e}_1, \mathbf{s} \rangle + \langle \mathbf{b}, \mathbf{r} \rangle + e_2) + \mu \right\rfloor \pmod{t} \\ &= \left\lfloor \frac{t}{q} (\langle \mathbf{A}^\top \cdot \mathbf{s} + \mathbf{b}, \mathbf{r} \rangle + \langle \mathbf{e}_1, \mathbf{s} \rangle + e_2) + \mu \right\rfloor \pmod{t} \\ &= \mu + \left\lfloor \frac{t}{q} (\langle \mathbf{e}, \mathbf{r} \rangle + \langle \mathbf{e}_1, \mathbf{s} \rangle + e_2) \right\rfloor \pmod{t}. \end{aligned}$$

This is equal to  $\mu$  if and only if every coefficient of  $\langle \mathbf{e}, \mathbf{r} \rangle + \langle \mathbf{e}_1, \mathbf{s} \rangle + e_2$  is in the interval  $[-\frac{q}{2t}, \frac{q}{2t})$ . It concludes the proof.  $\square$

## 4.2 Specification of SMAUG.KEM

We finally introduce the key encapsulation mechanism **SMAUG.KEM** in Figure 7. **SMAUG.KEM** is designed following the Fujisaki-Okamoto transform with implicit rejection using the non-perfectly correct public key encryption **SMAUG.PKE**. The construction of **SMAUG.KEM** involves the use of the following symmetric primitives:

- Hash function  $H$  for hashing a public key,
- Hash function  $G$  for deriving a sharing key and a seed.

The Fujisaki-Okamoto transform used in Figure 7 defers from the  $\text{FO}_m^\chi$  transform in [44] in encapsulation and decapsulation. When generating the sharing key and randomness, **SMAUG's Encap** utilizes the hashed public key, which prevents certain multi-target attacks. As for **Decap**, if  $\text{ct} \neq \text{ct}'$  holds, an alternative sharing key should be re-generated not to leak failure information against Side-Channel Attacks (SCA). However, even when the failure information is leaked, security can still rely on the explicit FO transform  $\text{FO}_m^\perp$ , recently treated in [42] with a competitive bound.

We also remark that the randomly chosen message  $\mu$  should be hashed additionally in the environments using a non-cryptographic system Random Number Generator (RNG). Using a True Random Number Generator (TRNG) is recommended to sample the message  $\mu$  in such devices.

We now show the completeness of **SMAUG.KEM** based on the completeness of the underlying public key encryption scheme, **SMAUG.PKE**.

|   |  |
|---|--|
| <b>KeyGen(<math>1^\lambda</math>):</b><br>1: $(pk, sk') \leftarrow \text{SMAUG.PKE.KeyGen}(1^\lambda)$<br>2: $d \leftarrow \{0, 1\}^{256}$<br>3: <b>return</b> $pk, sk = (sk', d)$  |  |
| <b>Encap(<math>pk</math>):</b><br>1: $\mu \leftarrow \{0, 1\}^{256}$<br>2: $(K, \text{seed}) \leftarrow G(\mu, H(pk))$<br>3: $ct \leftarrow \text{SMAUG.PKE.Enc}(pk, \mu; \text{seed})$<br>4: <b>return</b> $ct, K$   | $\triangleright pk = (\text{seed}_A, b)$ |
| <b>Decap(<math>sk, ct</math>):</b><br>1: $\mu' = \text{SMAUG.PKE.Dec}(sk', ct)$<br>2: $(K', \text{seed}') \leftarrow G(\mu', H(pk))$<br>3: $ct' = \text{SMAUG.PKE.Enc}(pk, \mu'; \text{seed}')$<br>4: <b>if</b> $ct \neq ct'$ <b>then</b><br>5: $(K', \cdot) \leftarrow G(d, H(ct))$<br>6: <b>return</b> $K'$ | $\triangleright sk = (sk', d)$           |

Fig. 7: Description of SMAUG.KEM

**Theorem 3 (Completeness of SMAUG.KEM).** *We borrow the notations and assumptions from Theorem 2 and Figure 7. Then SMAUG.KEM in Figure 7 is also  $(1 - \delta)$ -correct. That is, for every key-pair  $(pk, sk)$  generated by  $\text{KeyGen}(1^\lambda)$ , the shared keys  $K$  and  $K'$  are identical with probability larger than  $1 - \delta$ .*

*Proof.* The shared keys  $K$  and  $K'$  are identical if the decryption succeeds. Assuming the pseudorandomness of the hash function  $G$ , the probability of being  $K \neq K'$  can be bounded by the DFP of SMAUG.PKE. The completeness of SMAUG.PKE (Theorem 2) concludes the proof.  $\square$

### 4.3 Security proof

When proving the security of the KEMs constructed using FO transform in the (Q)ROM, one typically relies on the generic reductions from one-wayness or IND-CPA security of the underlying PKE. In the ROM, SMAUG.KEM has a tight reduction from the IND-CPA security of the underlying PKE, SMAUG.PKE. However, like other lattice-based constructions, the underlying PKE has a chance of decryption failures, which makes the generic reduction unapplicable [52] or non-tight [13, 42, 44] in the QROM. Therefore, we prove the IND-CCA security of SMAUG.KEM based on the non-tight QROM reduction of [13] as explained in Section 2 by proving the IND-CPA security of SMAUG.PKE.

**Theorem 4 (IND-CPA security of SMAUG.PKE).** *Assuming pseudorandomness of the underlying sampling algorithms, the IND-CPA security of SMAUG.PKE can be tightly reduced to the decisional MLWE and MLWR problems. Specifically, for any IND-CPA-adversary  $\mathcal{A}$  of SMAUG.PKE, there exist adversaries  $\mathcal{B}_0$ ,*

$\mathcal{B}_1$ ,  $\mathcal{B}_2$ , and  $\mathcal{B}_3$  attacking the pseudorandomness of XOF, and the pseudorandomness of sampling algorithms, the hardness of MLWE, and the hardness of MLWR, respectively, such that,

$$\begin{aligned} \text{Adv}_{\text{SMAUG.PKE}}^{\text{IND-CPA}}(\mathcal{A}) &\leq \text{Adv}_{\text{XOF}}^{\text{PR}}(\mathcal{B}_0) + \text{Adv}_{\text{expandA,HWT,dGaussian}}^{\text{PR}}(\mathcal{B}_1) \\ &\quad + \text{Adv}_{n,q,k,k}^{\text{MLWE}}(\mathcal{B}_2) + \text{Adv}_{n,p,q,k+1,k}^{\text{MLWR}}(\mathcal{B}_3). \end{aligned}$$

The secret distribution terms omitted in the last two advantages (of  $\mathcal{B}_1$  and  $\mathcal{B}_2$ ) are uniform over ternary polynomials with Hamming weights  $h_s$  and  $h_r$ , respectively. The error distribution term omitted in the advantage of  $\mathcal{B}_2$  is a pseudorandom distribution following the corresponding CDT.

*Proof.* The proof proceeds by a sequence of hybrid games from  $G_0$  to  $G_4$  defined as follows:

- $G_0$ : the genuine IND-CPA game,
- $G_1$ : identical to  $G_0$ , except that the public key is changed into  $(\mathbf{A}, \mathbf{b})$ ,
- $G_2$ : identical to  $G_1$ , except that the sampling algorithms are changed into truly random samplings,
- $G_3$ : identical to  $G_2$ , except that  $\mathbf{b}$  is randomly chosen from  $\mathcal{R}_q^k$ ,
- $G_4$ : identical to  $G_3$ , except that the ciphertext is randomly chosen from  $\mathcal{R}_p^k \times \mathcal{R}_{p'}$ . As a result, the public key and the ciphertexts are truly random.

We denote the advantage of the adversary on each game  $G_i$  as  $\text{Adv}_i$ , where  $\text{Adv}_0 = \text{Adv}_{\text{SMAUG.PKE}}^{\text{IND-CPA}}(\mathcal{A})$  and  $\text{Adv}_4 = 0$ . Then, it holds that

$$|\text{Adv}_0 - \text{Adv}_1| \leq \text{Adv}_{\text{XOF}}^{\text{PR}}(\mathcal{B}_0),$$

for some adversary  $\mathcal{B}_0$  against the pseudorandomness of the extendable output function. Given that the only difference between the transcripts viewed in hybrid games  $G_1$  and  $G_2$  is the randomness sampling, it can be concluded that

$$|\text{Adv}_1 - \text{Adv}_2| \leq \text{Adv}_{\text{expandA,HWT,dGaussian}}^{\text{PR}}(\mathcal{B}_1),$$

for some adversary,  $\mathcal{B}_1$  attacking the pseudorandomness of at least one of the samplers. The difference in the games  $G_2$  and  $G_3$  is in the way the polynomial vector  $\mathbf{b}$  is sampled. In  $G_2$ , it is sampled as part of an MLWE sample, whereas in  $G_3$ , it is randomly selected. Thus, the difference in the advantages  $\text{Adv}_2$  and  $\text{Adv}_3$  can be bounded by  $\text{Adv}_{n,q,k,k}^{\text{MLWE}}(\mathcal{B}_2)$ , where  $\mathcal{B}_2$  is an adversary distinguishing the MLWE samples from random. In the hybrids  $G_3$  and  $G_4$ , the only difference is in the way the ciphertexts are generated; they are either randomly chosen from  $\mathcal{R}_p^k \times \mathcal{R}_{p'}$  or generated to be  $(\mathbf{c}_1, \lfloor p'/p \cdot c_2 \rfloor)$ , where

$$\begin{bmatrix} \mathbf{c}_1 \\ c_2 \end{bmatrix} = \left\lfloor \frac{p}{q} \cdot \begin{pmatrix} \mathbf{A} \\ \mathbf{b}^\top \end{pmatrix} \cdot \mathbf{r} \right\rfloor + \frac{p}{t} \cdot \begin{bmatrix} 0 \\ \mu \end{bmatrix}.$$

If an adversary  $\mathcal{A}$  can distinguish the two ciphertexts, we can construct an adversary  $\mathcal{B}_3$  distinguishing the MLWR sample from random: *for given a sample*

$(\mathbf{A}, \mathbf{b}) \in \mathcal{R}_q^{(k+1) \times k} \times \mathcal{R}_p^{k+1}$ ,  $\mathcal{B}_3$  rewrites  $\mathbf{b}$  as  $(\mathbf{b}_1, b_2) \in \mathcal{R}_p^k \times \mathcal{R}_p$ , computes  $(\mathbf{b}_1, \lfloor p'/p \cdot b_2 \rfloor)$ , and use  $\mathcal{A}$  to decide the ciphertext type. The output of  $\mathcal{A}$  will be the output of  $\mathcal{B}_3$ . Therefore, we can conclude the proof by observing that

$$|\text{Adv}_3 - \text{Adv}_4| \leq \text{Adv}_{n,p,q,k+1,k}^{\text{MLWR}}(\mathcal{B}_3).$$

□

The classical IND-CCA security of SMAUG.KEM is then obtained directly from FO transforms [42] in the classical random oracle model. Theorem 1 implies the quantum IND-CCA security of SMAUG.KEM in the quantum random oracle model.

## 5 Parameter selection and concrete security

In this section, we first give a concrete security analysis of SMAUG and provide the recommended parameter sets.

### 5.1 Concrete security estimation

We exploit the best-known lattice attacks to estimate the concrete security of SMAUG.

**5.1.1 Core-SVP methodology.** Most of the known attacks are essentially finding a nonzero short vector in Euclidean lattices, using the Block–Korkine–Zolotarev (BKZ) lattice reduction algorithm [22, 41, 53]. BKZ has been used in various lattice-based schemes [3, 16, 31, 34, 55]. The security of the schemes is generally estimated as the time complexity of BKZ in core-SVP hardness introduced in [5]. It depends on the block size  $\beta$  of BKZ reporting the best performance. According to Becker et al. [10] and Chailloux et al. [21], the  $\beta$ -BKZ algorithm takes approximately  $2^{0.292\beta+o(\beta)}$  and  $2^{0.257\beta+o(\beta)}$  time in the classical and quantum setting, respectively. The polynomial factors and  $o(\beta)$  terms in the exponent are ignored. We use the lattice estimator [2] to estimate the concrete security of SMAUG in core-SVP hardness.

**5.1.2 Beyond Core-SVP methodology.** In addition to lattice reduction attacks, we also take into consideration the cost of other types of attacks, e.g., algebraic attacks like the Arora-Ge attack or Coded-BKW attacks, and their variants. In general, these attacks have considerably higher costs and memory requirements compared to previously introduced attacks.

We also focus on the attacks not considered in the lattice estimator, specifically those that target sparse secrets, such as Meet-LWE [51] attack. This attack is inspired by Odlyzko’s Meet-in-the-Middle approach and involves using representations of ternary secrets in additive shares. The asymptotic attack complexity is claimed as  $\mathcal{S}^{0.25}$ ; however, it is far from the estimated attack costs in SMAUG parameter sets. Even, the estimated cost has a significant gap with the real attack, due to the hidden costs behind the estimation.

We summarize the costs of the algebraic and combinatorial attacks in Table 2. Attack costs for Arora-Ge and Coded-BKW are estimated with lattice estimator [2]. The estimated cost of Arora-Ge attack on SMAUG 256 is not determined by lattice-estimator, outputting  $\infty$ , which is at least a thousand bits of security. The costs for the Meet-LWE attack are estimated with a python script<sup>6</sup> based on May’s analysis [51], best among Rep-1 and Rep-2.

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<sup>6</sup> The script can be found on the team SMAUG website: <http://kpqc.cryptolab.co.kr/>

|                                   |       |           |           |           |
|-----------------------------------|-------|-----------|-----------|-----------|
| Parameters sets                   |       | SMAUG 128 | SMAUG 192 | SMAUG 256 |
| Target security                   |       | 1         | 3         | 5         |
| Classical core-SVP                |       | 120.0     | 181.7     | 264.5     |
| Algebraic & Combinatorial attacks |       |           |           |           |
| Arora-Ge                          | time  | 741.3     | 983.4     | -         |
|                                   | (mem) | (598.0)   | (636.5)   | -         |
| BKW                               | time  | 144.7     | 202.0     | 274.6     |
|                                   | (mem) | (133.7)   | (190.7)   | (256.9)   |
| Meet-LWE                          | time  | 164.3     | 213.8     | 283.2     |
|                                   | (mem) | (143.7)   | (192.4)   | (254.7)   |

Table 2: Attack costs beyond Core-SVP.

**5.1.3 MLWE hardness.** We estimated the cost of the best-known attacks for MLWE, including *primal attack*, *dual attack*, and their hybrid variations, in the core-SVP hardness. We remark that any  $\text{MLWE}_{n,q,k,\ell,\eta}$  instance can be viewed as an  $\text{LWE}_{q,nk,n\ell,\eta}$  instance. Although the MLWE problem has an additional algebraic structure compared to the LWE problem, no attacks currently take advantage of this structure. Therefore, we assess the hardness of the MLWE problem based on the hardness of the corresponding LWE problem. We also consider the distributions of secret and noise when estimating the concrete security of SMAUG. We have also analyzed the costs of recent attacks that aim to target the MLWE problem with sparse secrets. Our narrow discrete Gaussian sampler’s tail bound is considered in estimating the security using the lattice estimator.

**5.1.4 MLWR hardness.** To measure the hardness of the MLWR problem, we treat it as an MLWE problem since no known attack utilizes the deterministic error term in the MLWR structure. Banerjee et al. [9] provided the reduction from the MLWE problem to the MLWR problem, which was subsequently improved in [6, 7, 15]. Basically, for given an MLWR sample  $(\mathbf{A}, \lfloor p/q \cdot \mathbf{A} \cdot \mathbf{s} \rfloor \bmod p)$  with uniformly chosen  $\mathbf{A} \leftarrow \mathcal{R}_q^k$  and  $\mathbf{s} \leftarrow \mathcal{R}_p^\ell$ , it can be expressed as  $(\mathbf{A}, p/q \cdot (\mathbf{A} \cdot \mathbf{s} \bmod q) + \mathbf{e} \bmod p)$ . The MLWR sample can be converted to an MLWE sample over  $\mathcal{R}_q$  by multiplying  $q/p$  as  $(\mathbf{A}, \mathbf{b} = \mathbf{A} \cdot \mathbf{s} + q/p \cdot \mathbf{e} \bmod q)$ . Assuming that the error term in the resulting MLWE sample is a random variable, uniformly distributed within the interval  $(-q/2p, q/2p]$ , we can estimate the hardness of the MLWR problem as the hardness of the corresponding MLWE problem.

## 5.2 Parameter sets

The SMAUG is parameterized by various integers such as  $n, k, q, p, p', t, h_s$  and  $h_r$ , as well as a standard deviation  $\sigma > 0$  for the discrete Gaussian noise. Our main focus when selecting these parameters is to minimize the ciphertext size while maintaining security. We set SMAUG parameters to make SMAUG at least as safe as Saber. We first set our ring dimension to  $n = 256$  and plaintext modulus to  $t = 2$  to have a 256-bit message space (or sharing key space). Then we search for

parameters that offer the smallest ciphertext size with enough security. Starting from parameters having a tiny ciphertext size, we increase the ciphertext size,  $h_s$ ,  $h_r$ , and  $\sigma$  and search for the parameters having enough security. Once we have them, we compute the DFP. If it is enough low, we can choose the compression parameter  $p'$ , but if it is not, we continue searching for appropriate parameters. The compression factor  $p'$  can be set to a small integer if the DFP is low enough. Else, we can keep  $p' = 256$  as in the level-3 parameter, and not compress the ciphertext.

Table 3 shows the three parameter sets of **SMAUG**, corresponding to NIST’s security levels 1, 3, and 5. For security levels 3 and 5, we can not find the parameters for  $q = 1024$ , so we use  $q = 2048$ . Especially, the standard deviation  $\sigma = 1.0625$  is too low for security level 3, so we move to  $\sigma = 1.453713$ . For the level-5 parameters set, we use  $k = 5$  since  $k = 4$  is too small for enough security.

| Parameters sets<br>Security level | SMAUG-128<br>1     | SMAUG-192<br>3      | SMAUG-256<br>5     |
|-----------------------------------|--------------------|---------------------|--------------------|
| $n$                               | 256                | 256                 | 256                |
| $k$                               | 2                  | 3                   | 5                  |
| $(q, p, p', t)$                   | (1024, 256, 32, 2) | (2048, 256, 256, 2) | (2048, 256, 64, 2) |
| $(h_s, h_r)$                      | (140, 132)         | (198, 151)          | (176, 160)         |
| $\sigma$                          | 1.0625             | 1.453713            | 1.0625             |
| Classical core-SVP                | 120.0              | 181.7               | 264.5              |
| Quantum core-SVP                  | 105.6              | 160.9               | 245.2              |
| Beyond core-SVP                   | 144.7              | 202.0               | 274.6              |
| DFP                               | -119.6             | -136.1              | -167.2             |
| Secret key                        | 176                | 236                 | 218                |
| Public key                        | 672                | 1088                | 1792               |
| Ciphertext                        | 672                | 1024                | 1472               |

Table 3: The NIST security level, selected parameters, classical and quantum core-SVP hardness and security beyond core-SVP (see Section 5.1.2), DFP (in  $\log_2$ ), and sizes (in bytes) of **SMAUG**.

The core-SVP hardness is estimated via the lattice estimator [2] using the cost model “ADPS16” introduced in [5] and “MATZOV” [50]. In the table, the smaller cost is reported. We assumed that the number of 1s is equal to the number of  $-1$ s for simplicity, which conservatively underestimates security.

The security beyond core-SVP is estimated via the lattice estimator [2] and the Python script implementing the Meet-LWE attack cost estimation. It shows the lowest attack costs among coded-BKW, Arora-Ge, and Meet-LWE attack and their variants. We note that these attacks require a minimum memory size of  $2^{130}$  to  $2^{260}$ .

### 5.3 Decryption failure probability

As our primary goal is to push the efficiency of the lattice-based KEMs toward the limit while keeping roughly the same level of security, we follow the frameworks given in the NIST finalist Saber. In particular, we set the DFP to have similar to or lower than that of Saber’s.

The impact of DFP on the security of KEM is still being investigated. However, we can justify our decision to follow Saber’s choice and why it is sufficient for real-world scenarios. To do this, we make the following assumptions:

1. Each key pair has a limit of  $Q_{\text{limit}} = 2^{64}$  decryption queries, as specified in NIST’s proposal call.
2. There are approximately  $2^{33}$  people worldwide, each with hundreds of devices. Each device has hundreds of *usable* public keys broadcasted for KEM.
3. We introduce an observable probability and assume it is far less than  $2^{-20}$ . Even though the decryption failure occurs, it can only be used for an attack when observed. Attackers can observe it through a side-channel attack, which enables the observation of decapsulation failures in the mounted device, or through direct communications after key derivation, allowing the detection of decryption failures with a communication per key pair. We assume the two cases can occur much less than  $2^{-20}$ , as they require physically mounted devices or communications with shared keys.

Based on these assumptions, we can deduce that the number of observable decryption failures can be upper bounded by  $2^{64+33+8+8} \cdot 2^{-20} = 2^{93}$ . Based on the best-known (multi-target) attacks for Saber [27, Figure 6 (a)], the quantum cost for finding a single failing ciphertext of SMAUG security level 3 is much higher than  $2^{160}$ , as desired<sup>7</sup>. For security level 5, we refer to Figure 7(a) in [27], which shows that the quantum cost for finding a single failure is much higher than  $2^{245}$ . Regardless of the attack cost estimated above, the scenario of checking the failures in more than  $2^{40}$  different devices is already way too far from the real-world attack scenario.

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<sup>7</sup> Specifically, the number of observable failures must be larger than  $1/\beta$  in [27] to observe at least one failing ciphertext. That is,  $\beta$  should be larger than  $2^{93}$ . The quantum cost is given as  $1/\beta\sqrt{\alpha}$ .



## 6 Implementation

In this section, we give the implementation performance for each parameter set. We compare the sizes and the reported performance with prior works such as Kyber and Saber. The constant-time reference implementation of SMAUG, along with the supporting scripts, can be found in our website: [www.kpqc.cryptolab.co.kr/smaug](http://www.kpqc.cryptolab.co.kr/smaug), github: [www.kpqc.cryptolab.co.kr/smaug](https://github.com/kpqc/smaug).

### 6.1 Performance

We instantiate the hash functions  $G$ ,  $H$  and the extendable output function XOF with the following symmetric primitives:  $G$  is instantiated with SHAKE256,  $H$  is instantiated with SHA3-256, and XOF is instantiated with SHAKE128.

Table 4 presents the performance results of SMAUG. For a fair comparison, we also performed measurements on the same system with identical settings of the reference implementation of Kyber and Saber<sup>8</sup>. All benchmarks are obtained on one core of an Intel(R) Core(TM) i7-10700K CPU processor with clock speed 3.80GHz. The benchmarking machine has 64 GB of RAM and runs Debian GNU/Linux with Linux kernel version 5.4.0. The implementation is compiled with gcc version 9.4.0, and the compiler flags as indicated in the CMakeLists included in the submission package.

| Schemes    | Cycles |        |        | Cycles (ratio) |       |       |
|------------|--------|--------|--------|----------------|-------|-------|
|            | KeyGen | Encap  | Decap  | KeyGen         | Encap | Decap |
| Kyber512   | 131560 | 162472 | 18930  | 1.8            | 2.27  | 2.08  |
| LightSaber | 93752  | 122176 | 133764 | 1.29           | 1.71  | 1.47  |
| SMAUG-128  | 72949  | 71418  | 91007  | 1              | 1     | 1     |
| Kyber768   | 214160 | 251308 | 285378 | 1.58           | 2.03  | 1.83  |
| Saber      | 18722  | 224686 | 239590 | 1.38           | 1.82  | 1.53  |
| SMAUG-192  | 135753 | 123763 | 156293 | 1              | 1     | 1     |
| Kyber1024  | 332470 | 371854 | 415498 | 1.39           | 1.61  | 1.54  |
| FireSaber  | 289278 | 347900 | 382326 | 1.21           | 1.51  | 1.41  |
| SMAUG-256  | 239689 | 230455 | 270365 | 1              | 1     | 1     |

Table 4: Median cycle counts of 1000 executions for Kyber, Saber, and SMAUG (and their ratios). Cycle counts are obtained on one core of an Intel Core i7-10700k, with TurboBoost and hyperthreading disabled, using the C implementations without AVX optimizations.

### 6.2 Security against physical attacks

We justify the security of SMAUG against physical attacks based on the profiled Differential Power Analysis (DPA). Specifically, Simple Power Analysis (SPA)

<sup>8</sup> From [github.com/pq-crystals/kyber](https://github.com/pq-crystals/kyber) (518de24) and [github.com/KULeuven-COS-IC/SABER](https://github.com/KULeuven-COS-IC/SABER) (f7f39e4), respectively.

can profile the key generation and encapsulation processes since they only occur once or without a secret key. However, multi-trace attacks are possible for decapsulation due to “re-encryption.” As Kyber and Saber share many design aspects with **SMAUG**, we can follow recent works on masking Kyber and Saber [11, 18] to add SCA countermeasures. Our new sampler, **dGaussian**, is expressed with bit operations, so adding SCA countermeasures like boolean masking is easy. While Krausz et al. [47] have recently proposed masking methods for the fixed hamming weight sampler, their efficiency is lacking, so we see it as future work. The new multiplication method may be vulnerable to memory access patterns, but we can efficiently mask it using coefficient-wise shuffling.

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